

Assessing Supply Chain Risk with Few Compulsory Subcontractors

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Abstract

In this study we propose a supply chain risk analytical model that incorporates three compulsory subcontractors, which are structured in a two-layer formation, while fault recognition by the monitoring chief manufacturer might be delayed. We analyze this precise production line since it simultaneously covers both a sole mandatory supplier and a second production layer, which includes two subcontractors that can partially back up each other throughout the production. This specific configuration not only can assist future investigations in building different quantitative schemes, but this formation also is rather widespread across the semiconductor industry. We therefore present a current illustrative example from this sector. In addition, we authenticate the validity of the proposed model and examine several sensitivities within through multiple numerical simulations. We find that the expected time to the next production failure is mostly sensitive to output variations within the sole supplier, and least sensitive to temporary pauses in the information flow across the supply network.

JEL Classifications: C63, D24

Keywords: supply chain risk, production disruption, compulsory subcontractors, fault recognition, chief manufacturer

1. Introduction

Over the past decade, numerous scholars have shown mounting interest in defining, classifying, and exploring the roots of Supply Chain Risk (SCR). In general, SCRs evolve from manufacturers' inability to generate products on time and deliver those goods to the final assembly line at the required quantity or quality. These production problems habitually cause a failure to meet customers' demand or even compromise safety provisos thus can further trigger financial, performance, physical, reputation, social, and time losses to the parties involved.

In this study we offer a novel analytical SCR model that incorporates three compulsory subcontractors, which are structured in a two-layer formation, while fault recognition by the monitoring chief manufacturer might be delayed. We construct this particular production line since, in essence, it simultaneously covers both a sole mandatory supplier as well as a second production layer, which includes two subcontractors that can back up each other throughout the production, at least to some degree. It is not only that this specific configuration can assist future investigations in building different SCR quantitative schemes, but also it appears that this formation is rather widespread across the semiconductor industry. We therefore present a current illustrative example

from this sector. Furthermore, we authenticate the validity of the proposed model and examine several model sensitivities through multiple numerical simulations.

Throughout history, there are numerous legendary examples of significant supply chain disruptions. Among others we recollect that in 1998, a computer glitch at Ford's supplier of door and trunk latches resulted in a three-day shutdown of the Fiesta and Puma manufacturing facilities in Germany. These three days alone cost Ford nearly 70 million pounds and the production of roughly 7,000 vehicles. Other instances include Ericsson that lost approximately 400 million euros after its supplier's semiconductor plant caught on fire in 2000. Ford closed again five major factories for several days after all air traffic was suspended shortly after the September 11, 2001 terror attack in the U.S. Land Rover laid off 1,400 employees when one of its subcontractors filed for bankruptcy in 2001. More recently, Dell had to recall four million hazardous laptop batteries made by Sony in 2006.

Ordinarily, there are multiple sources of SCR. Disruptions to production could take place because of output variability and time inconsistency across the supply chain, natural disasters like earthquakes, flood, fire, or volcano eruptions, wars and terrorism, non-compliance to production specifications, sudden liquidation of subcontractors, complications for multinationals in foreign exchange rates, management failures such as fraud or dishonesty, violations of protection rights or intellectual property laws, or other contractual impediments with subcontractors. The implications of SCR are vast. These largely include the risk of inadequate inventory, price escalation, layoffs, legal disruptions, loss of reputation, or other financial consequences.

The importance of studying and developing quantitative SCR models naturally escalates due to several current industry trends. These modern universal tendencies include the rise in the globalization of markets, the increase in strategic outsourcing by worldwide manufacturers, the growth in the reliance on specialized suppliers (mostly among technological firms), the expansion of global search for competitive advantages, and the emergence of information technologies that assist in monitoring complex supply networks.

In light of these recent global trends and because production hazards often have an enormous impact on corporate profitability, we deploy hereafter a quantitative model that assists supply chain managers in gauging the likelihoods of future production disruptions. These prognostic estimations may direct production managers to better concentrate on the critical points throughout the manufacturing line. Risk auditors can also integrate the computed failure probabilities suggested here with the likely cost estimations at their specific production lines and through that assess the comprehensive distribution of probable manufacturing expenses.

The study proceeds as follows. In Section 2 we provide a relatively short yet representative survey of the recent literature on SCR. In Section 3 we outline our SCR model with few compulsory subcontractors and possible delay to the information flow. In Section 4 we illustrate the validity of the proposed scheme with a genuine example from the semiconductor industry. In Section 5 we enhance intuition on the matter by simulating the main derivations and exploring several parameter sensitivities. In Section 6 we conclude.

2. Recent Literature on SCR

To place the current investigation in the right context and to further accentuate our unique contribution we survey hereafter some recent studies that discuss principal issues of SCR. Due to the immense volume of relevant literature, the following list of references is naturally incomplete. Nonetheless, these selected articles, composed from the last ten years, shall provide the readers sufficient knowledge on the current topic and motivate our analytical scheme.

Harland, Brenchley, and Walker (2003) present a review of definitions and classifications of the types of risks that jointly assemble SCR. Among the various dimensions impacting supply networks the authors identify scale, technological novelty, quantity of sub-systems components, degree of customization within the final product or service, quantity of alternative design and delivery paths, number of feedback loops throughout the production line and delivery arrangements, variety of distinct knowledge bases, skills and competencies incorporated, intensity and extent of end user involvement, uncertainty of end user requirements, extent of supplier involvement in the innovation and transformation process, regulatory environment, number of agents in the supply network, complexity of the financial contracts supporting the production, and extent of political and stakeholder intervention. Zsidisin (2003) examines several case studies of setbacks within supply chains and recognizes that the roots of SCR usually evolve from individual supplier failures as well as from market factors. The individual supplier failures largely include the inability to handle demand variations, the incompetence to provide quality products, and the incapacity to stay in pace with technological innovations. Market determinants mostly contain protective patent registrations and other market capacity constraints.

Bogataj and Bogataj (2004) explore two key problems of SCR by seeking optimal facilities' location and ideal production capacity given to these facilities to achieve the best response time when market perturbations are feasible. Christopher and Lee (2004) comment that SCR has intensified over the years since product and technology life-cycles have shortened significantly and competitive product introductions make life-cycle demand difficult to predict. The authors add that improving end-to-end visibility among subcontractors could mitigate SCR. Faisal, Banwet, and Shankar (2006) provide a hierarchy-based framework to reduce SCR by understanding the mutual relationships among various enablers. In particular, the authors recommend that production managers devote their maximum attention to the group of enablers having a high driving power and low dependence.

Tang (2006) reviews several quantitative models for managing SCR and discusses the broad demand for new models in this field. Wagner and Bode (2006) further investigate the relationship between supply chain vulnerability and SCR by conducting a large-scale survey among 760 executives from firms operating in Germany. The authors report that supply chain characteristics such as a firm's dependence on certain customers and suppliers, the degree of single sourcing, and the reliance on global supply sources are the most dominant aspects of SCR. Bogataj and Bogataj (2007) propose a quantitative scheme of SCR that accommodates the costs in the event that certain products are not at the required location, at the expected time, and of the essential quality or quantity. Goh, Lim, and Meng (2007) present a stochastic model for the multi-stage global supply chain network problem while incorporating a set of related hazards, including supply, demand, exchange, and disruption risks.

Ritchie and Brindley (2007) suggest a different perspective on SCR by integrating the dimensions of supply and performance sources, drivers, consequences, and management responses and further provide a categorization of the respective risk drivers. Manuj and Mentzer (2008) focus on global SCR by surveying the literature and deploying a qualitative study based on 14 interviews and a focus-group meeting with senior supply chain executives of multinationals. Schoenherr, Tummala, and Harrison (2008) inspect the process used by a U.S. manufacturing company to assess SCR within the context of an offshore sourcing decision and empirically derive and cluster 17 relevant risk factors into main-objectives and sub-objectives. Main-objectives include sourcing characteristics related to the product, the partner, and the environment. Sub-objectives are further divided into product aspects and partner aspects. Product aspects include quality and cost, and partner aspects comprise service and management capabilities.

Tang and Tomlin (2008) offer a unified framework and five stylized analytical models that delve into the open question of how much flexibility is really needed to mitigate SCR. Wu and Olson (2008) first display three SCR evaluation models: a Chance Constrained Programming (CCP)

model, a Data Envelopment Analysis (DEA) model, and a Multi-Objective Programming (MOP) model. The authors then form a hypothetical supply chain consisting of three levels and use simulated data with representative distributions to contrast and discuss the advantages and disadvantages of these three types of SCR evaluation schemes. Neiger, Rotaru, and Churilov (2009) present a novel value-focused process engineering methodology for process-based SCR identification with the overall intention to increase value to the individual supply chain members as well as to the entire supply chain.

Oke and Gopalakrishnan (2009) categorize SCR into inherent high-frequency risks and disruption infrequent risks. While the high-likelihood risks usually exhibit low impact on supply chains, the low-likelihood risks typically convey high impact on supply networks. The authors further examine both generic and specific mitigation strategies for dealing with these exposures. Sodhi and Tang (2009) explain that companies normally manage their SCR either with the strategic long-term or the tactical medium-term strategies. The authors then employ a linear programming model of deterministic supply chain planning that considers demand uncertainty and cash flows within a medium term. Trkman and McCormack (2009) present a new conceptual approach to identify and predict supply chain disruptions based on prior classification of problematic suppliers with respect to their attributes, performances, and supply chain characteristics, which are further modified by environmental factors.

Yu, Zeng, and Zhao (2009) evaluate the impacts of supply disruption risks on the choice between the famous single and dual sourcing methods in a two-stage supply chain with a non-stationary and price-sensitive demand. Tang and Musa (2011) survey the literature on SCR management and identify three main disruption flows in supply networks, namely material (products or components), cash (financial arrangements), and information (transparency across subcontractors). The authors state that quantitative models in this field are relatively scarce while information flow risk has received inadequate attention thus far. Thun and Hoenig (2011) review 67 manufacturing plants in the German automotive industry and further test the impact of SCR management on corporate performance while dividing the sample into two groups: the firms that take reactive steps and those that undertake preventative measures. The authors report that the group using reactive SCR management has higher average value in terms of disruptions resilience, whereas the group pursuing preventative SCR management has better values concerning production flexibility.

By reviewing these prior studies as well as examining past instances we learn that SCR typically surface within two dimensions: (1) the presence of few compulsory subcontractors that cannot deliver on time necessary components to the main production line, while (2) any delay in the detection of these supply chain problems could add further assembly or delivery costs. In light of these two aspects, we direct our model hereafter to account for the failure likelihoods of limited mandatory suppliers and the time-delay of fault recognition by the chief manufacturer.

3. A Model with Few Compulsory Subcontractors

We now turn to develop an explicit SCR model that accounts for a chief manufacturer, which relies on three subcontractors structured in a two-layer formation, while fault recognition by the chief manufacturer could be delayed, depending on the nature of disruption and the degree of information flow between all the system's producers. In particular, we assume that a chief manufacturer i outsources the production of two components vital to its lead product among three subcontractors. While supplier j is a sole authorized subcontractor that is exclusively responsible to assemble one of the critical components, suppliers k and l share production capacity within the second crucial module. Nevertheless, subcontractors k and l , which operate at a separate production layer than supplier j , are not necessarily autonomous. Without loss of generality, we assume that a production

failure by any one of these two suppliers could trigger an immediate shift in the Production Failure Rate (PFR) of the other. This realistic presumption considers that upon a recognition of a production failure of either subcontractors k or l , more production responsibilities will be instantaneously assigned to the remaining supplier, hence the residual subcontractor within this manufacturing layer would have to increase its production speed to be able to service the present customers' demand. This enhanced production rate would conceivably intensify the respective PFR.

We therefore denote α_k the initial PFR of subcontractor k , α_l the original PFR of subcontractor l , β_k the new PFR of supplier k upon a prior failure of subcontractor l , β_l the new PFR of supplier l upon a prior failure of subcontractor k , and δ_j the independent PFR of the sole supplier j . Alternatively, we can assert that $\beta_k = C_k \alpha_k$ and $\beta_l = C_l \alpha_l$, where $C_k \geq 1$ and $C_l \geq 1$ are two constants corresponding to the suppliers' shifts from normal production speeds to enhanced production rates.

In addition, we assume that the chief manufacturer i intermittently monitors the entire production line including the outsourced assembly of parts that are manufactured by the three subcontractors j , k , and l . Thus, to minimize its response time, the chief manufacturer aspires to instantly detect any production disruption at its suppliers. However, we realistically presume that this coverage is imperfect hence fault recognition by the chief manufacturer could be delayed due to unintentional temporary problems in the information flow between the three suppliers and the main contractor, or because of intentional actions taken by the subcontractors to mask these infrequent production interruptions. We therefore designate γ_i the Fault Recognition Rate (FRR) by the chief manufacturer. All rates thus far can also be interpreted as probabilities per time unit.

For the purpose of model tractability we also consider that throughout the production horizon all the relevant parameters, α_k , α_l , β_k , β_l , δ_j , and γ_i are constants. This presumption allows us to allocate average quantities or expected value to the above PFR and FRR and at the same time to further calibrate the model with every consequential change to these rates. In addition, we assume that all time intervals between production failures as well as fault recognitions are exponentially distributed. Not only that the Exponential distribution serves in most related disciplines a parallel purpose, but this specific selection conveys an added value in our context. The Exponential distribution is a continuous dissemination, thus it allows us to yield reduced-form solutions, which are further programmable. This apparent advantage is highly beneficial when modeling complex production systems.¹

The supply chain described thus far exhibits several risks, as follows. Whenever sole supplier j fails to provide its designated parts, the chief manufacturer cannot complete the production of its lead product. Alternatively, when both subcontractors k and l fail to supply their nominated components, the entire production line comes to a halt as well. Nevertheless, the production process is considered fully operational if sole supplier j is working effectively and at least one of the subcontractors k or l is functional, despite having a higher PFR upon a prior failure of the complement supplier within the same production layer. Clearly, if all three subcontractors are perfectly operative, the entire production runs smoothly.

We can further describe this specific supply chain network with a set of three sufficient and necessary conditions. The production line labeled thus far is fully functional at time τ hence no apparent production disruption occurs until time τ if and only if any one of the following three mutually exclusive States of Nature (SN) happens:

¹ The Weibull distribution is another popular allocation when modeling failure rates. Although a deployment of the Weibull distribution requests a few more underlying assumptions for justifying the shape and the scale parameters, with relatively little effort, the current scheme can utilize this substitute dissemination when needed.

SN_1 : All subcontractors j , k , and l are perfectly operative by time τ .

SN_2 : Sole supplier j has not failed by time τ , supplier k fails at time ξ where $0 \leq \xi \leq \tau$, fault recognition is instantaneously achieved by the chief manufacturer i at time ξ , and supplier l continues to be operative (despite its enhanced production load) during the time interval $[\xi, \tau]$.

SN_3 : Sole supplier j has not failed by time τ , supplier l fails at time ξ where $0 \leq \xi \leq \tau$, fault recognition is instantaneously achieved by the chief manufacturer i at time ξ , and supplier k continues to be operative (despite its enhanced production load) during the time interval $[\xi, \tau]$.

In this setting, since the above three states of nature are indeed mutually exclusive, the inclusive likelihood for a fully functional supply chain at time τ can be expressed as either a union or a sum of the marginal probabilities, as follows:

$$\Omega(\tau) = \cup_{m=1}^3 P(SN_m) = \sum_{m=1}^3 P(SN_m). \quad (1)$$

Under the assumption of exponentially distributed production failure rates and fault recognition rate we can now disclose the marginal probability for the first state of nature as:

$$P(SN_1) = \prod_{j,k,l} [1 - \Phi(\tau; \lambda)] = [1 - (1 - e^{-\delta_j \tau})][1 - (1 - e^{-\alpha_k \tau})][1 - (1 - e^{-\alpha_l \tau})] = e^{-\delta_j \tau} e^{-\alpha_k \tau} e^{-\alpha_l \tau} = e^{-(\delta_j + \alpha_k + \alpha_l) \tau}, \quad (2)$$

where $\Phi(\tau; \lambda)$ represents the Cumulative Distribution Function (CDF) of the PFR and FRR at time τ with general respective rates λ . Furthermore, we can obtain the marginal probability for the second state of nature as:

$$P(SN_2) = \int_0^\tau e^{-\delta_j \tau} (\alpha_k e^{-\alpha_k \xi}) e^{-\alpha_l \xi} e^{-\gamma_i \xi} e^{-\beta_l (\tau - \xi)} d\xi = \alpha_k e^{-(\delta_j + \beta_l) \tau} \int_0^\tau e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_l) \xi} d\xi = \frac{\alpha_k e^{-(\delta_j + \beta_l) \tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_l) \tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_l}, \quad (3)$$

while the term $\alpha_k e^{-\alpha_k \xi}$ naturally represents the Probability Density Function (PDF) for a production failure within supplier k at time ξ . The third state of nature is almost perfectly analogous to the second, excluding a slight modification of the assignments given to subcontractors k and l , hence the marginal probability for the third state of nature is:

$$P(SN_3) = \frac{\alpha_l e^{-(\delta_j + \beta_k) \tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_k) \tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_k}. \quad (4)$$

We shall notice that both derivations (3) and (4) are mathematically well defined, even in the rare case where $\alpha_k + \alpha_l + \gamma_i = \beta_l$ in the denominator of equation (3), or within the singular event of $\alpha_k + \alpha_l + \gamma_i = \beta_k$ in the denominator of equation (4). When $\alpha_k + \alpha_l + \gamma_i - \beta_l = 0$ in derivation (3), we can utilize L'Hopital's rule and attain

$$P(SN_2) = \frac{\alpha_k e^{-(\delta_j + \beta_l) \tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_l) \tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_l} = \alpha_k e^{-(\delta_j + \beta_l) \tau} \lim_{\psi \rightarrow 0} \frac{1 - e^{-\psi \tau}}{\psi} = \alpha_k e^{-(\delta_j + \beta_l) \tau} \lim_{\psi \rightarrow 0} \frac{d(1 - e^{-\psi \tau})/d\psi}{d\psi/d\psi} = \alpha_k e^{-(\delta_j + \beta_l) \tau} \frac{\tau e^{-\psi \tau}}{1} \Big|_{\psi=0} = \alpha_k e^{-(\delta_j + \beta_l) \tau} \tau, \quad (5)$$

where we define a temporary variable $\psi := \alpha_k + \alpha_l + \gamma_i - \beta_l$. Similarly, when $\alpha_k + \alpha_l + \gamma_i - \beta_k = 0$ in derivation (4), we get

$$P(SN_3) = \frac{\alpha_l e^{-(\delta_j + \beta_k)\tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_k)\tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_k} = \alpha_l e^{-(\delta_j + \beta_k)\tau} \lim_{\eta \rightarrow 0} \frac{1 - e^{-\eta\tau}}{\eta} = \alpha_l e^{-(\delta_j + \beta_k)\tau} \tau, \quad (6)$$

where we define a second temporary variable $\eta := \alpha_k + \alpha_l + \gamma_i - \beta_k$.

We can now integrate results (2), (3), and (4) into equation (1) and acquire the total probability for a fully functional supply chain at time τ as:

$$\Omega(\tau) = e^{-(\delta_j + \alpha_k + \alpha_l)\tau} + \frac{\alpha_k e^{-(\delta_j + \beta_l)\tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_l)\tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_l} + \frac{\alpha_l e^{-(\delta_j + \beta_k)\tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_k)\tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_k}. \quad (7)$$

To illustrate how supply chain managers could further calibrate the model with different prognostic conventions, we wish to explore several intriguing specifications here, which yield minimal reduced form solutions, as follows. First, when both suppliers k and l do not adjust their PFR upon a prior failure of the other subcontractor within the same production layer, hence when both suppliers k and l are considered statistically independent of one another, i.e. $\alpha_k = \beta_k$ and $\alpha_l = \beta_l$, then equation (7) develops into:

$$\Omega(\tau) = e^{-(\delta_j + \alpha_k + \alpha_l)\tau} + \frac{\alpha_k e^{-(\delta_j + \alpha_l)\tau} [1 - e^{-(\alpha_k + \gamma_i)\tau}]}{\alpha_k + \gamma_i} + \frac{\alpha_l e^{-(\delta_j + \alpha_k)\tau} [1 - e^{-(\alpha_l + \gamma_i)\tau}]}{\alpha_l + \gamma_i}. \quad (8)$$

Second, when both suppliers k and l are administered by the same initial PFR, hence $\alpha_k = \alpha_l = \alpha$, and since $\beta_k = C_k \alpha_k$ and $\beta_l = C_l \alpha_l$, where $C_k \geq 1$ and $C_l \geq 1$, if in addition $C_k = C_l = C$, then equation (7) becomes:

$$\Omega(\tau) = e^{-(\delta_j + 2\alpha)\tau} + \frac{2\alpha e^{-(\delta_j + C\alpha)\tau} \{1 - e^{-(2-C)\alpha + \gamma_i}\tau}\}}{(2-C)\alpha + \gamma_i}. \quad (9)$$

Particularly, if $C = 2$ then we can further reduce derivation (9) to:

$$\Omega(\tau) = e^{-(\delta_j + 2\alpha)\tau} + \frac{2\alpha e^{-(\delta_j + 2\alpha)\tau} (1 - e^{-\gamma_i\tau})}{\gamma_i}. \quad (10)$$

Third, when the production line is divided only between two statistically independent subcontractors having the same PFR, hence in the special case where $\alpha_k = \alpha_l = \beta_k = \beta_l = \mu$ and $\delta_j = \gamma_i = 0$ then equation (7) transforms to:

$$\begin{aligned} \Omega(\tau) &= e^{-2\mu\tau} + 2e^{-\mu\tau}(1 - e^{-\mu\tau}) = \\ &= e^{-2\mu\tau} + 2e^{-\mu\tau} - 2e^{-2\mu\tau} = 2e^{-\mu\tau} - e^{-2\mu\tau}. \end{aligned} \quad (11)$$

We delineate these distinct instances to demonstrate the large flexibility of the proposed scheme to handle various scenarios of diverse supply chains. In addition, at this stage of the analysis we desire to attain the most probable time to the next production disruption. We recall that the time-related likelihood for a production failure is the complement to the probability of a fully functional production line. More formally, we write

$$\Theta(\tau) := \Omega^c(\tau) = 1 - \Omega(\tau), \quad (12)$$

where $\Theta(\tau)$ denotes the production failure time distribution function, and $\theta(\tau) = \frac{\partial\Theta(\tau)}{\partial\tau}$ represents the production failure time density function. In this situation, the Expected Time to Production Disruption (ETPD) is:²

$$\begin{aligned}
 ETPD &= \int_0^\infty \tau \theta(\tau) d\tau = \int_0^\infty \tau \left[\frac{\partial\Theta(\tau)}{\partial\tau} \right] d\tau = - \int_0^\infty \tau \left[\frac{\partial\Omega(\tau)}{\partial\tau} \right] d\tau = \int_0^\infty \Omega(\tau) d\tau = \\
 &= \int_0^\infty e^{-(\delta_j + \alpha_k + \alpha_l)\tau} + \frac{\alpha_k e^{-(\delta_j + \beta_l)\tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_l)\tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_l} + \frac{\alpha_l e^{-(\delta_j + \beta_k)\tau} [1 - e^{-(\alpha_k + \alpha_l + \gamma_i - \beta_k)\tau}]}{\alpha_k + \alpha_l + \gamma_i - \beta_k} d\tau = \\
 &= \int_0^\infty e^{-(\delta_j + \alpha_k + \alpha_l)\tau} d\tau + \\
 &= \frac{1}{\delta_j + \alpha_k + \alpha_l} + \left[\frac{\alpha_k}{\alpha_k + \alpha_l + \gamma_i - \beta_l} \left(\frac{1}{\delta_j + \beta_l} - \frac{1}{\alpha_k + \alpha_l + \gamma_i + \delta_j} \right) \right] + \left[\frac{\alpha_l}{\alpha_k + \alpha_l + \gamma_i - \beta_k} \left(\frac{1}{\delta_j + \beta_k} - \frac{1}{\alpha_k + \alpha_l + \gamma_i + \delta_j} \right) \right].
 \end{aligned} \tag{13}$$

We can further tune this outcome to handle the special cases discussed earlier. When both suppliers k and l do not adjust their PFR upon a prior disruption at the other subcontractor within the same production layer, hence when $\alpha_k = \beta_k$ and $\alpha_l = \beta_l$, then equation (13) becomes:

$$ETPD = \frac{1}{\delta_j + \alpha_k + \alpha_l} + \left[\frac{\alpha_k}{\alpha_k + \gamma_i} \left(\frac{1}{\delta_j + \alpha_l} - \frac{1}{\alpha_k + \alpha_l + \gamma_i + \delta_j} \right) \right] + \left[\frac{\alpha_l}{\alpha_l + \gamma_i} \left(\frac{1}{\delta_j + \alpha_k} - \frac{1}{\alpha_k + \alpha_l + \gamma_i + \delta_j} \right) \right]. \tag{14}$$

On the other hand, when both suppliers k and l are influenced by the same initial PFR, hence $\alpha_k = \alpha_l = \alpha$, and when $\beta_k = \beta_l = 2\alpha$, then equation (13) turns into:

$$ETPD = \frac{1}{\delta_j + 2\alpha} + \left[\frac{\alpha}{\gamma_i} \left(\frac{2}{\delta_j + 2\alpha} - \frac{2}{2\alpha + \gamma_i + \delta_j} \right) \right]. \tag{15}$$

Furthermore, when the production capacity is divided only between two statistically independent subcontractors having the same PFR, hence when $\alpha_k = \alpha_l = \beta_k = \beta_l = \mu$ and $\delta_j = \gamma_i = 0$, then equation (13) develops into:

$$ETPD = \frac{1}{2\mu} + \frac{2}{\mu} - \frac{1}{\mu} = \frac{3}{2\mu}. \tag{16}$$

4. Illustrative Example

After only a few years of experience, many consider the latest generations of the Apple iPad to be among the leading tablet computers in the market. The first iPad was released on April 3, 2010, while it launched several design precedents, including screen size and button placement. The second iPad product added front-facing and rear-facing video cameras. The third generation iPad complemented the unique Apple design with a Retina display, which is a liquid crystal screen having an excessive pixel density, and the fourth iPad upgraded towards the end of 2012 a powerful processor.

² Balagurusamy (1984, p. 58) elaborates on the asymptotic behavior of a system that can be described by its reliability function and a general hazard model.

While these advanced technical features have cleared the way for Apple Inc. to become one of the primary technological companies these days, they also created some robust dependencies throughout the production line. According to several media sources, in March 2012, Apple Inc. decided to lean on a sole supplier for its Retina displays. After LG and Sharp failed to meet Apple's distinguished quality standards, Samsung became the sole supplier of Retina displays for the latest generations of iPad. In addition, some of the iPad's internal computer chips are exclusively supplied by two of the largest semiconductor manufacturers, Intel and TSMC. Both manufacturers supplement the production of the iPad's next processor, named the A7 chip, among other components. The binding contracts with these three subcontractors, however, may confront not only common production jeopardies but also some unique SCR.

The first distinctive SCR considers the sole supplier Samsung. Over the past years, Apple and Samsung have filed numerous lawsuits against one another across several countries. These litigations mostly surround the misuse of various patents and could potentially lead to significant production disruptions in the future. The second uncommon SCR relates to TSMC, which resides in a different production layer than Samsung, since it shares manufacturing responsibilities with Intel on some specified computer chips. To date, all 15 fabs (semiconductor production facilities) of TSMC are in Taiwan, which is located in a seismologically active region. Taiwan is an island exposed to numerous periodic earthquakes, where sporadic major tremors could cause significant production delays due to the need to recalibrate the highly sensitive machineries at the fabs after strong seismic vibrations.³

Production disruptions at any of these three subcontractors could have substantial implications towards the production of iPad at the chief manufacturer Apple Inc., as well as trigger significant financial losses for Apple's shareholders and creditors.⁴ The monetary consequences of either production delays or production flaws could be devastating to all parties involved.⁵ Thus, production managers must assess the likelihoods of these SCRs in advance. We therefore provide in the following section more insight into this customary supply network by simulating the proposed model with ample realistic quantities.

5. Simulations

Since we are unaware of any accessible database that may provide us real assessments of PFR of subcontractors or FRR of chief manufacturers, we wish to theoretically simulate the model's behavior by using presumably genuine figures. We therefore assume hereafter that most suppliers operate flawlessly throughout the vast majority of time and that chief manufacturers tend to closely monitor the entire production line.⁶ Thus, we deploy relatively low PFR and FRR within the main simulation as well as throughout the successive four notional recreations, which are meant to explore various sensitivities to the model parameters.

³ On September 21, 1999, a major earthquake killed 2,415 people within Nantou County, Taiwan. This exceptionally strong tremor also injured 11,305 individuals, and further caused a \$10 billion worth of damage throughout the area.

⁴ Historically, Apple Inc. was a debt-free enterprise. Nonetheless, on April 30, 2013, Apple sold the largest corporate-bond deal in history, \$17 billion issuance.

⁵ Such a production flaw already occurred during 2012 with the iPhone 4's overheating problem and battery drain.

⁶ In addition, across all simulations we constantly preserve the realistic relations at which $\alpha_k \leq \beta_k$ and $\alpha_l \leq \beta_l$.

In **Table 1** we examine the general performance of the proposed SCR model by assigning the following fixed quantities: $\alpha_k = 0.04$, $\alpha_l = 0.06$, $\delta_j = 0.05$, $\beta_k = 0.09$, $\beta_l = 0.12$, $\gamma_i = 0.08$, and $\tau = \{0, 1, \dots, 12\}$ months. For these specific values, we observe that the time-related total probability for a fully functional supply chain $\Omega(\tau)$ from equation (7) gradually decays from one at origin to 0.292 after only twelve months. This outcome suggests a notable reduction of more than 70% in the overall reliability of the production line, despite fairly low PFR and FRR per time unit. Furthermore, we find that the *ETPD* from equation (13) resides at 9.55 months, detachedly from the simulated progress in the time indicator τ . We further utilize these pivot values in the following simulations while we alternate each parameter at the time.

In **Table 2** we test how the *ETPD* shifts once we modify the PFR α_l within the interval $[0.01, 0.11]$. In **Table 3** we inspect how the *ETPD* transforms when we alter the PFR β_k in the closed range $[0.05, 0.15]$. Without loss of generality, these two sensitivity analyses assess the overall conduct of the system with gradual increments in the PFR within the production layer that has two supernumerary yet correlated subcontractors. In **Table 4** we check how the *ETPD* adjusts when we change the PFR δ_j within the domain of $[0.01, 0.11]$. This complement investigation focuses on the production layer that has a single mandatory supplier. These three latter experiments show that, as predicted by the model, the *ETPD* is inversely related to all PFR, hence the projected time to the next system failure decreases with every increase in the PFR of the subcontractors. Moreover, we realize that the steepest decline in the *ETPD* occurs as a result of a reduction in the isolated PFR δ_j . This result is highly intuitive, since subcontractor j is a sole compulsory supplier, while subcontractors k and l back up each other, at least to some extent.

In **Table 5** we further analyze how the *ETPD* adapts when we revise the FRR γ_i of the chief manufacturer within the interval $[0.01, 0.11]$. We find that the *ETPD* decreases from 10.82 months at initiation to 9.22 months after a year. We therefore conclude that although some reduction in the *ETPD* is observed due to the surge in the FRR γ_i , this time delay, which corresponds to disruptions in the information flow throughout the supply chain, is somewhat subordinate, compared to the other model variables. The FRR γ_i by the chief manufacturer has a relatively minor impact on the overall performance of this production configuration.

Table 1. Model Simulation with Pivot Numbers

α_k	α_l	δ_j	β_k	β_l	γ_i	τ	$\Omega(\tau)$	<i>ETPD</i>
0.04	0.06	0.05	0.09	0.12	0.08	0	1.000	9.55
0.04	0.06	0.05	0.09	0.12	0.08	1	0.943	9.55
0.04	0.06	0.05	0.09	0.12	0.08	2	0.877	9.55
0.04	0.06	0.05	0.09	0.12	0.08	3	0.807	9.55
0.04	0.06	0.05	0.09	0.12	0.08	4	0.736	9.55
0.04	0.06	0.05	0.09	0.12	0.08	5	0.666	9.55
0.04	0.06	0.05	0.09	0.12	0.08	6	0.599	9.55
0.04	0.06	0.05	0.09	0.12	0.08	7	0.536	9.55
0.04	0.06	0.05	0.09	0.12	0.08	8	0.478	9.55
0.04	0.06	0.05	0.09	0.12	0.08	9	0.424	9.55
0.04	0.06	0.05	0.09	0.12	0.08	10	0.376	9.55
0.04	0.06	0.05	0.09	0.12	0.08	11	0.332	9.55
0.04	0.06	0.05	0.09	0.12	0.08	12	0.292	9.55

In this simulation we assign constant values to the model parameters, as follows: $\alpha_k = 0.04$, $\alpha_l = 0.06$, $\delta_j = 0.05$, $\beta_k = 0.09$, $\beta_l = 0.12$, $\gamma_i = 0.08$, while allowing the time indicator to vary between $\tau =$

{0, 1, ..., 12} months. We then measure the consequent progress in the time-related inclusive probability for a fully functional supply chain $\Omega(\tau)$ from equation (7). In addition we record the autonomous projected time to the next production disruption *ETPD* from equation (13).

Table 2. Model Sensitivity to the Production Failure Rate (PFR) α_l

α_k	α_l	δ_j	β_k	β_l	γ_i	τ	$\Omega(\tau)$	<i>ETPD</i>
0.04	0.01	0.05	0.09	0.12	0.08	0	1.000	11.70
0.04	0.02	0.05	0.09	0.12	0.08	1	0.946	11.08
0.04	0.03	0.05	0.09	0.12	0.08	2	0.885	10.58
0.04	0.04	0.05	0.09	0.12	0.08	3	0.816	10.17
0.04	0.05	0.05	0.09	0.12	0.08	4	0.742	9.84
0.04	0.06	0.05	0.09	0.12	0.08	5	0.666	9.55
0.04	0.07	0.05	0.09	0.12	0.08	6	0.590	9.31
0.04	0.08	0.05	0.09	0.12	0.08	7	0.516	9.11
0.04	0.09	0.05	0.09	0.12	0.08	8	0.446	8.93
0.04	0.10	0.05	0.09	0.12	0.08	9	0.383	8.78
0.04	0.11	0.05	0.09	0.12	0.08	10	0.326	8.65

In this simulation we designate the following quantities to the model parameters: $\alpha_k = 0.04$, $\alpha_l \in \{0.01, 0.11\}$, $\delta_j = 0.05$, $\beta_k = 0.09$, $\beta_l = 0.12$, $\gamma_i = 0.08$, while allowing the time indicator to advance across $\tau = \{0, 1, \dots, 12\}$ months. We first measure the consequent progress in the projected time to the next production disruption *ETPD* from equation (13). The realized reduction in the *ETPD* is independent of the time display hence it is strictly associated with the enhancement in the PFR α_l . For purpose of robustness, we further assess the total probability per time-unit for a fully functional supply chain $\Omega(\tau)$ from equation (7), which does rely on τ .

Table 3. Model Sensitivity to the Production Failure Rate (PFR) β_k

α_k	α_l	δ_j	β_k	β_l	γ_i	τ	$\Omega(\tau)$	<i>ETPD</i>
0.04	0.06	0.05	0.05	0.12	0.08	0	1.000	10.30
0.04	0.06	0.05	0.06	0.12	0.08	1	0.944	10.06
0.04	0.06	0.05	0.07	0.12	0.08	2	0.879	9.86
0.04	0.06	0.05	0.08	0.12	0.08	3	0.809	9.70
0.04	0.06	0.05	0.09	0.12	0.08	4	0.736	9.55
0.04	0.06	0.05	0.10	0.12	0.08	5	0.663	9.43
0.04	0.06	0.05	0.11	0.12	0.08	6	0.592	9.32
0.04	0.06	0.05	0.12	0.12	0.08	7	0.524	9.22
0.04	0.06	0.05	0.13	0.12	0.08	8	0.460	9.14
0.04	0.06	0.05	0.14	0.12	0.08	9	0.401	9.06
0.04	0.06	0.05	0.15	0.12	0.08	10	0.348	8.99

In this simulation we designate the following quantities to the model parameters: $\alpha_k = 0.04$, $\alpha_l = 0.06$, $\delta_j = 0.05$, $\beta_k \in \{0.05, 0.15\}$, $\beta_l = 0.12$, $\gamma_i = 0.08$, while allowing the time indicator to advance across $\tau = \{0, 1, \dots, 12\}$ months. We first measure the consequent progress in the projected time to the next production disruption *ETPD* from equation (13). The realized reduction in the *ETPD* is autonomous of the time scale hence it is exclusively related to the enhancement in the PFR β_k . For purpose of robustness, we further assess the total probability per time-unit for a fully functional supply chain $\Omega(\tau)$ from equation (7), which does rely on τ .

Table 4. Model Sensitivity to the Production Failure Rate (PFR) δ_j

α_k	α_l	δ_j	β_k	β_l	γ_i	τ	$\Omega(\tau)$	<i>ETPD</i>
0.04	0.06	0.01	0.09	0.12	0.08	0	1.000	13.87
0.04	0.06	0.02	0.09	0.12	0.08	1	0.972	12.49
0.04	0.06	0.03	0.09	0.12	0.08	2	0.913	11.34
0.04	0.06	0.04	0.09	0.12	0.08	3	0.832	10.38
0.04	0.06	0.05	0.09	0.12	0.08	4	0.736	9.55
0.04	0.06	0.06	0.09	0.12	0.08	5	0.634	8.84
0.04	0.06	0.07	0.09	0.12	0.08	6	0.532	8.22
0.04	0.06	0.08	0.09	0.12	0.08	7	0.435	7.68
0.04	0.06	0.09	0.09	0.12	0.08	8	0.347	7.20
0.04	0.06	0.10	0.09	0.12	0.08	9	0.271	6.78
0.04	0.06	0.11	0.09	0.12	0.08	10	0.206	6.40

In this simulation we allocate the following numbers to the model parameters: $\alpha_k = 0.04$, $\alpha_l = 0.06$, $\delta_j \in \{0.01, 0.11\}$, $\beta_k = 0.09$, $\beta_l = 0.12$, $\gamma_i = 0.08$, while allowing the time indicator to change throughout $\tau = \{0, 1, \dots, 12\}$ months. We first measure the resulting progress in the projected time to the next production disruption *ETPD* from equation (13). The realized reduction in the *ETPD* is independent of the time indicator hence it is strictly correlated with the simulated increments in the PFR δ_j . For purpose of robustness, we further record the total probability per time-unit for a fully functional supply chain $\Omega(\tau)$ from equation (7), which does rely on τ .

Table 5. Model Sensitivity to the Fault Recognition Rate (FRR) γ_i

α_k	α_l	δ_j	β_k	β_l	γ_i	τ	$\Omega(\tau)$	<i>ETPD</i>
0.04	0.06	0.05	0.09	0.12	0.01	0	1.000	10.82
0.04	0.06	0.05	0.09	0.12	0.02	1	0.912	11.75
0.04	0.06	0.05	0.09	0.12	0.03	2	0.884	10.35
0.04	0.06	0.05	0.09	0.12	0.04	3	0.817	10.16
0.04	0.06	0.05	0.09	0.12	0.05	4	0.747	9.99
0.04	0.06	0.05	0.09	0.12	0.06	5	0.676	9.83
0.04	0.06	0.05	0.09	0.12	0.07	6	0.605	9.68
0.04	0.06	0.05	0.09	0.12	0.08	7	0.536	9.55
0.04	0.06	0.05	0.09	0.12	0.09	8	0.472	9.43
0.04	0.06	0.05	0.09	0.12	0.10	9	0.412	9.32
0.04	0.06	0.05	0.09	0.12	0.11	10	0.357	9.22

In this simulation we designate the following quantities to the model parameters: $\alpha_k = 0.04$, $\alpha_l = 0.06$, $\delta_j = 0.05$, $\beta_k = 0.09$, $\beta_l = 0.12$, $\gamma_i \in \{0.01, 0.11\}$, while allowing the time indicator to advance across $\tau = \{0, 1, \dots, 12\}$ months. We first measure the consequent progress in the projected time to the next production disruption *ETPD* from equation (13). The realized reduction in the *ETPD* is independent of the time display hence it is strictly associated with the enhancement in the FRR γ_i . For purpose of robustness, we further assess the total probability per time-unit for a fully functional supply chain $\Omega(\tau)$ from equation (7), which does rely on τ .

Overall, we perceive that the PFR δ_j of the sole supplier is the most influential parameter on the *ETPD*. This isolated PFR reduces the projected time to the next production disruption from 13.87 months to 6.40 months. The next influential parameter is the fundamental PFR α_l , which restrains

the *ETPD* from 11.70 months to 8.65 months. The impact spectrum is then followed by the consequent PFR β_k upon an earlier failure of subcontractor l within the same production layer, which reduces the projected time to the next production disruption from 10.30 months to 8.95 months. The least influential parameter is the FRR γ_i , which slightly moderates the *ETPD* from 10.82 months to 9.22 months. We therefore conclude that production managers should devote their attention to possible disruptions within the supply chain with priorities sorted according to this listing.

For purpose of robustness, we have replicated these five simulations with other realistic quantities. Nonetheless, throughout these supplementary experiments we have not noticed any materially different patterns, thus we report here only the results from the core simulations.

6. Summary and Conclusions

Supply chain risks can be principally defined as the potential financial or reputational losses transferred to manufacturers due to temporary discrepancies between supply, or production output, and customers' demand in terms of either inadequate quantity or quality. It is well documented throughout modern history that production disruptions mostly occur with the presence of few compulsory suppliers, while information flow problems across the supply network typically intensify the already grave consequences of these manufacturing breaks.

In the current study we suggest a general analytical scheme that assesses the probabilities of manufacturing disruptions in different production layers and further integrate these likelihoods into a comprehensive measure of the chances for a production failure. We incorporate two types of subcontractors, a sole mandatory supplier and a binding pair of correlated suppliers, which can somewhat back up each other upon interim production disturbances. In addition, we account for a possible time-delay for the chief manufacturer to recognize a production fault. We therefore provide a diagnostic formula for the total probability of a fully functional supply chain at any given time as well as an estimation of the projected time until the next production disruption.

We illustrate this common supply chain with a genuine anecdote concerning the production line of Apple iPad and its dependency on the sole supplier, Samsung, for the unique Retina display and on the two alternative semiconductor manufacturers, Intel and TSMC, for other iPad electronic modules. We further test the sensitivity of the concluding derivations to the relevant parameters and deduce that, as intuitively predicted, the expected time to the next production failure is mostly sensitive to output variations within the sole supplier, and least sensitive to temporary pauses in the information flow across the supply network.

Future studies of supply chain risk can naturally build upon the current investigation. Not only can we treat the present model as a pivotal scheme, which can be calibrated to fit various production scenarios, but also the conventional mathematical procedures deployed here can serve later scholars in assembling other forms of analytical models of supply chain risk.

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