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Credit Portfolio Risk Evaluation based on the Pair Copula VaR Models

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Abstract

Aiming to solve the difficulty in describing the high dimensional dependency structure of credit assets, we construct pair copula VaR model to evaluate the credit portfolio risk. The empirical study which takes the publicly traded companies in Shanghai stock exchanges and Shenzhen stock exchanges shows that the Clayton copula with Canonical vine structure is the most appropriate function to describe the high dimensional low tail dependency structure. Meanwhile, the Monte Carlo simulation result proves that the pair copula VaR model can accurately measure the credit portfolio risk both in calm period and crisis period. Additionally, we acquired the optimal weights of the different credit assets in portfolio according to the simulation results of pair copula VaR models. Based on the research results, the commercial banks can dynamically adjust their credit asset allocation, so as to alleviate the credit portfolio risk and conduct more efficient credit risk management.

JEL Classifications: G21

Keywords: Pair Copula VaR; Low tail dependency; Credit portfolio risk; Canonical vine structure; D vine structure; Monte Carlo simulation

1. Introduction

Due to the credit risk contagion and the financial crisis, recent studies emphasize more on the credit portfolio management, rather from the individual perspectives. On the one hand, being influenced by some systemic factors, such as the recession of macroeconomics or the external crisis shocks, credit quality of enterprises may decline simultaneously. On the other hand, credit risk easily contagion through the channel of the supply chain or the capital linkage. Under this background, the evaluation bias could be caused if the commercial banks simply and linearly aggregate the individual enterprises' credit risk, and it essential to accurately measure the portfolio risk of credit asset. While evaluating the credit portfolio risk, it is difficult to grasp the nonlinear and low tail dependency of the credit assets, especially for the multi-assets.

This paper attempts to describe the nonlinear and low tail dependency of credit asset portfolio by using the Pair Copula VaR model, and thus provide a new method to evaluate the credit portfolio risk. The empirical study which takes the listed companies in Shanghai Stock Exchange and Shenzhen Exchange as the research samples shows that the canonical vine structure copula model can capture the low tail dependency of credit assets compared to other structure copula models, and pair copula VaR models can not only get the maximum loss of the credit portfolio, but also acquire the optimal weights of different assets in the portfolio. Therefore, the results are helpful for the risk management of the commercial bank and supervisors.

The rest of the paper is organized as follows: Section 2 reviews the representative survey of the recent literature on credit portfolio management. Section 3 provides the specific procedures of modeling the pair copula VaR models. In Section 4 we estimate the parameters of the pair copula models, and compare the different vine structure of pair copula models by using the financial data of Chinese listed companies. Section 5 presents the result of pair copula VaR of the sample commercial bank. Concluding remarks and management implications are presented in Section 6.

2. Literature Review

Since the credit risk is easy to transmit among different enterprises, credit portfolio management has been paid more and more attention by the risk managers and scholars. When an enterprise defaults, the default probability of other enterprises may increase (Weigel & Gemmill, 2006; Allen & Carletti, 2006; Takada & Sumita, 2011; Huang & Cheng, 2013; Boudreault, Gauthier, & Thomassin, 2014). Under the framework of credit portfolio management, adjusting the loan weights of different industries can change the overall risk of the credit portfolio (Bangia, Diebold, Kronimus, Schagen, & Schuermann, 2002; Frey & McNeil, 2002; Dietsch & Petey, 2002; Schlottmann & Seese, 2004; Nyström & Skoglund, 2006; Horst, 2007; Ebnöther & Vanini, 2007; Huang & Oosterlee, 2009; Pra & Tolotti, 2009; Rosen & Saundeers, 2010; Chan & Kroese, 2010; Tsaig, Levy, & Wang, 2011; Choe & Jang, 2011; Iscoe, Kreinin, Mausser, & Romanko, 2012; Lu, Huang, Ching, & Siu, 2013; Liang, Wang, & Dong, 2013; Li, Wang, & Wang, 2013).

For the purpose of optimizing the credit assets allocation and decreasing the overall risk of the credit portfolio, Frey and McNeil (2002) develop VaR(Value at Risk, VaR) to estimate the loss of credit portfolio under a certain level of confidence, thereafter, Dietsch and Petey(2002) provide a VaR measures in a portfolio under the framework of New Ratings-Based Basel Capital Accord. Schlottmann and Seese (2004) design a non-convex Credit-Value-at-Risk downside risk measure which is relevant to real world credit portfolio. Iscoe *et al*. (2012) use two realistic credit portfolios to assess the in-and-out-of-sample performance for the resulting VaR and ES optimized portfolios. Li *et al*. (2013) apply the factor copula model to calculate the credit VaR (value at risk) for the target portfolio. Although the above literatures use different VaR methods to measure the credit assets portfolio risk of different samples, they all find that the VaR method is an effective way to evaluate the credit portfolio risk.

When calculating the VaR of the credit portfolio, it is critical important to measure the credit risk correlations of different credit assets in the portfolio. The correlation structure has an important influence in measuring the risk and the distribution of credit portfolio loss. Since credit risk often has an obvious fat tail characteristic (Lucas, Klaassen, Spreij, & Straetmans, 2001; Rosenberg & Schuermann, 2006; Pagnoncelli & Cifuentes, 2014), thus the linear correlation that most existing literatures use may lead to inaccurate of evaluation result. Meanwhile, the linear correlation is difficult to describe the asymmetry dependency structure, and some literatures have proved that the shocks to correlation is different between the advantageous and disadvantageous information or events, therefore, it is not appropriate to use linear correlation to evaluate the credit portfolio risk.

Trying to solve the above problem, the copula method is applied to evaluate the nonlinear correlation of the credit assets. Copula method, which has a rapid development recently, has statistical advantages. Copulas are functions that link univariate distributions to the multivariate distribution of the related variables, and allow us to construct a flexible multivariate distribution without the choice of marginal distribution, thus the copula method provides an effective way to describe the various dependency structures. Li (2000) uses Gauss Copula function to price credit derivatives, later, most scholars use copulas to price the corporate debt (Melchiori, 2003; Das & Geng, 2006; Hamerle & Rösch, 2005; Hull & White, 2006; Crook & Moreira, 2011). Some scholars also apply the copula method to evaluate the credit portfolio risk. Chan & Kroese (2010) use t-Copula function to study the possibility of large-cap group's loss, and they find that the t-Copula function is more sensitive to tail correlation. Liu (2011) provides coherent measures of credit portfolio risk based on t-copula. Crook and Moreira (2011) use binary and static Copula method to model the asymmetric default risk correlation of credit card. Choe and Jang (2011) use multiple Archimedes' Copula method to price the Basket Default Swap and verify the effectiveness of their algorithm. Iscoe *et al*. (2012) and Li *et al*. (2013) calculate the VaR of credit portfolio on the basis of measuring credit correlation structure by using different kinds of Copula function.

Copula method has become an important way to measure the credit portfolio risk and optimize the structure of credit portfolio. Some literature have developed copula VaR to evaluate the credit portfolio risk and get a good application effect, while this method still need to be improved. Most of the existing literatures focus on static and binary copula, and this function setting is not consistent with the practice of the commercial banks' risk management activities, for the reason that the number of credit assets in a portfolio may be high dimensional. In our paper, we attempt to fill this gap between the existing literature and the risk management practice. In order to describe the asymmetry and low tail dependency of the credit assets, we apply pair copula to VaR framework, and use Monte Carlo simulation method to get the pair copula VaR, thus provide a new approach to measure the evaluate the credit portfolio risk.

3. Procedures of the Modeling the Pair Copula VaR

3.1. The Credit Portfolio Risk under the VaR Framework

Suppose the VaR of a certain credit portfolio is $\text{VaR}_{i+1}(\alpha) = \inf\{s{:}F_t(s) \geq \alpha\}$ under the confidence level of 1-α. Where F_t represents the distribution function of yield $X_{p,t}$ at the time t. It can be presented in terms of the probability measure as:

$$p(X_{p,i} \leq \text{VaR}_i(\alpha) \big| \Omega_{t-1}) = \alpha \tag{1}$$

Under the condition of the given information set Ω_{t-1} at the time (t—1) and at a given level of confidence, VaR equals to the maximum loss in a future time t. Considering a portfolio with n kinds of credit assets $x_{i,t}$, i=1, 2…,n, the return of this credit portfolio is: $X_{p,t}=\omega_1 x_{1,t}+\omega_2 x_{2,t} +,\dots,+ \omega_n x_{n,t} - EL$, Where ($\omega_1$, ω_2,…, ω_n) represents n kinds of assets' proportion in the portfolio. Where EL represents the expected loss:

$$EL = \sum_{i=1}^{n} w_i \times EAD_i \times PD_i \times RL_i \tag{2}$$

where EL is the function of w_i, EAD_i, PD_i, RL_i and R. n represents the kind of credit assets. Here w_i represents the weights of the credit asset i. EAD_i represents the Exposure at Default of the asset i. PD_i is the default probability of credit asset i. RL_i represents the loss given default. The VaR of credit portfolio could be represented as:

$$P\left(X_{p,t} \leq VaR_t(\alpha) \mid \Omega_{t-1}\right) = P\left(\sum_{i=1}^{n} P_i r_i - EL \leq VaR_t(\alpha) \mid \Omega_{t-1}\right) = \alpha \tag{3}$$

Where, P_i and r_i represent the credit portfolio and the interest rate of this portfolio. By formula (3), we can get the possible loss of the commercial bank, meanwhile, according to the possible loss, and the commercial banks can also acquire the optimal weights of the different credit assets.

3.2. The Calculation Steps of Measuring Credit Portfolio Risk Based On Pair Copula VaR Models

According to the above framework, we calculate the pair copula VaR of the credit portfolio risk throughout the following steps:

Step 1: Calculate the credit risk correlation of credit assets from different industries by using pair copula models.

Step 2: Estimate the LGD (Loss Given Default, LGD). Default probability of each company is determined by the internal rating, according to the default probability, the LGD can be estimated by the different types of the loans.

Step 3: Calculate the credit rating transition matrix. Using the credit risk's historical data, we calculate the probability that a certain credit rank transit to other credit ranks. .

Step 4: Simulate the credit changes of the commercial banks' credit portfolio. Using Monte Carlo simulation method, we simulate the credit changes as follows:

Firstly, we generate random sequences $\{\omega_i, i=1, 2, \ldots, n\}$ which obeys uniform distribution with the parameters$[a, b]$. The value of a and b is determined by the credit rating transition matrix. We define the indices range of the credit risk as rank g is $[g_c, g_d]$. For a specific industry, if the probability of keeping rank g in subsequent period is p_g and the time of transition is T, then we can generate random sequences $T\times P_g$ which also obey the uniform distribution.

Secondly, the simulation sequence $\{x_1, x_2,\ldots,x_n\}$ can be deduced by according to the conditional distribution function $\mathrm{F}(x_j|x_1, x_2, \ldots, x_{j-1})$. Refering Aas, Czado, Frigessi, and Bakken (2009), the distribution function is determined by the Copula distribution function $C(., \ldots , .)$ of different pair canonical decomposition.

We can determine the simulation sequence of joint distribution function which obeys the Pair Copula decomposition according to formula (4).

$$\begin{cases} x_1 = \omega_1 \\ x_2 = F_{2|1}^{-1}\left(\omega_2 \mid x_1\right) \\ x_3 = F_{3|2}^{-1}\left(\omega_3 \mid x_1, x_2\right) \\ \ldots\ldots \\ x_n = F_{n|1,2,\ldots\text{n-1}}^{-1}\left(\omega_n \mid x_1, x_2 \ldots x_{n-1}\right) \end{cases} \tag{4}$$

Step 5: Calculate the expected loss of credit assets portfolio. Based on the credit rating of each asset in credit portfolio, and the simulation result from step 4, we can acquire the expected loss of credit portfolio.

Step 6: For a given confidence level (1-α), the VaR of credit portfolio can be determined by the simulated expected losses and the formula $P(X_{p,t}\leq \mathrm{VaR}_\alpha)=\alpha$.

4. Parameters Estimation of the Pair Copula Models

4.1. Selection of the Benchmark Copula Function

For the purpose of measuring the credit risk correlation, the credit risk indexes should be constructed before the measurement. We combine the results of multivariate discriminant analysis, support vector machine model and KMV model by using minimum error variance method to assess the credit risk of the single enterprises, and the specific procedures is illustrated in Xie, Luo, and Yu (2011), Luo and Ouyang(2012). Then, industry credit risk indexes are calculated by using the debt to total debt of industry as the weights. The samples are publicly traded listed companies in Shanghai Stock Exchanges and Shenzhen Stock Exchanges. According to the clustering analysis results, four types of industries are selected as the final samples to estimate the parameters of the pair copula models. These four industries are production and supply of power, gas and water, wholesale and retail establishments, petroleum, chemical and plastics, and information technology, and these four industries are represented for strong cyclical industry, defense industry, weak cyclical industry and growth industry. We respectively use I, II, III and IV to stand for these four types of industries. The credit risk variation of the above four industries are shown in **Fig. 1**.

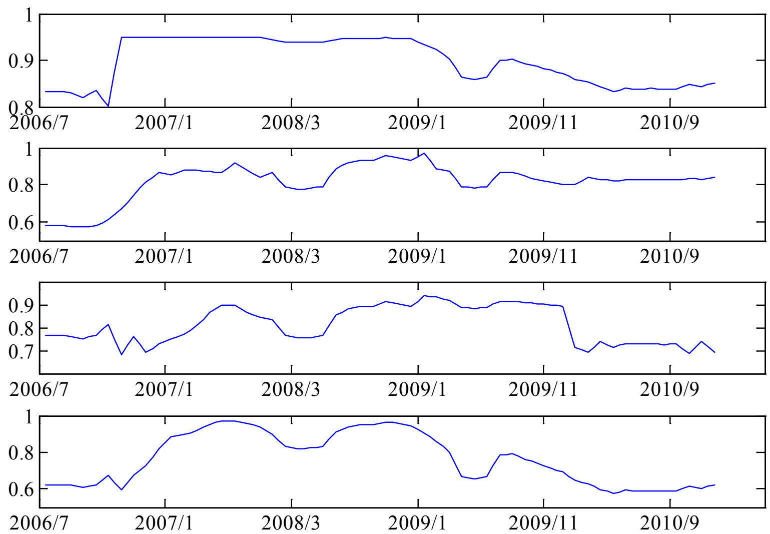


Fig.1. The Credit risk of different industries

We use empirical distribution function to transform the original data, thus all four sequences obey the uniform distribution, and thus the transformed data meet the basic requirement of the copula models. According to two step estimation procedure (Patton, 2006), we can estimate the parameters of binary Copula functions. Estimation results are shown in **Table 1**.

Table 1. The Parameters of the binary copula models

Copula	Parameters	I–II	I–III	I–IV	II–III	II–IV	III–IV
Normal Copula	ρ	0.556(1.194)	0.552(1.596)	0.742(1.893)	0.396(1.310)	0.621(1.727)	0.687(1.945)
	LR	86.728	57.648	114.537	97.641	98.167	117.564
t Copula	ρ	0.785(20.728)	0.727(18.496)	0.872(43.715)	0.859(1.051)	0.8981(28.155)	0.9160(46.742)
	n	2.100(3.229)	2.322(5.808)	2.100(8.502)	2.100(1.002)	2.100(1.183)	2.100(0.513)
	LR	105.05	81.98	151.43	111.03	144.44	181.01
SJC Copula	τ^U	0.299(1.630)	0.351(2.160)	0.581(4.354)	0.384(1.536)	0.517(2.742)	0.644(3.943)
	τ^L	0.609(3.319)	0.519(3.193)	0.663(4.971)	0.409(1.638)	0.5897(3.1255)	0.608(3.722)
	LR	66.835	53.6475	109.303	35.57	79.39	106.22
Gumbel Copula	τ^G	0.215(1.131)	0.194(1.112)	0.375(1.058)	1.178(1.249)	0.303(1.0321)	0.510(1.810)
	LR	59.912	54.51	94.94	22.17	84.80	118.46
Clayton Copula	τ^C	0.428(4.736)	0.382(4.247)	0.524(4.724)	0.326(2.740)	0.454(4.2744)	0.510(4.448)
	LR	60.64	66.73	95.07	26.63	66.92	86.79

Note: numbers in brackets are t-statistics.

The normal Copula function basically reflects the dependency of credit risk under the normal condition, and t-Copula function is sensitive to the change of low tail. The Gumbel Copula, Clayton Copula and SJC Copula functions can describe the upper and lower tail correlation of credit assets. Judging from the result in **Table 1**, we can conclude that there is a strong positive correlation between industry credit risks. Then, comparing the t-statistics of different copula models, we find that the parameters of t-Copula, SJC Copula and Clayton Copula are more significant than other copula functions, these result show that lower tail dependency are more suitable for describe the credit risk correlation. The results also suggest that credit correlation will increase when facing to unfavorable news and disadvantageous economic environment. Additionally, the empirical results above also show that Clayton Copula function is most preferable to describe credit risk correlation, therefore, this paper chooses Copula function as the benchmark function to decompose multivariate Copula function when constructing pair copula function. The expression of *n* dimension Clayton copula functions as shown as follow:

$$C(u_1, u_2, ..., u_N; \theta) = (\sum_{n=1}^{N} u_n^{-\alpha} - N + 1)^{-\frac{1}{\alpha}} \tag{5}$$

Where $\alpha \in \infty$ .When $\alpha \to 0$, the correlation coefficients of u are close to 0, and when $\alpha \to \infty$, these correlation coefficients are close to 1. Based on the existing literatures, the pair copula is usually decomposed by the Canonical vine structure and D vine structure. In our paper, we adopt these two types of decomposition structures, and compare the fitness of the different pair copula models, thus to find the optimal models to calculate the pair copula VaR for credit portfolio management.

4.2. Parameters Estimation of Pair Copula with Canonical Vine Structure

Under the Canonical vine decomposition structure, the joint density function of multivariate Copula function $f(x_1, x_2, x_3, x_4)$:

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = {} & f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot f(x_4) \\ & \cdot c_{12}(F(x_1), F(x_2)) \cdot c_{13}(F(x_1), F(x_3)) \cdot c_{14}(F(x_1), F(x_4)) \\ & \cdot c_{23|1}(F(x_2 | x_1), F(x_3 | x_1)) \cdot c_{24|1}(F(x_2 | x_1), F(x_4 | x_1)) \\ & \cdot c_{34|12}(F(x_3 | x_1, x_2), F(x_4 | x_1, x_2)) \end{aligned} \tag{6}$$

Each pair copula function can be decomposed into a product of binary copula density function and marginal distribution density function:

$$f(x | v) = c_{xv|v-j}(F(x | v_{-j}), F(v_j | v_{-j})) \cdot f(x | v_{-j}) \tag{7}$$

Then, we can calculate the conditional distribution function $F(x/v)$:

$$F(x | v) = \frac{\partial C_{xv_j|v_{-j}}(F(x | v_{-j}), F(v_j | v_{-j}))}{\partial F(v_j | v_{-j})} \tag{8}$$

The log-likelihood function of Canonical vine is:

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} \sum_{t=1}^{T} \log(c_{j,j+i|1,\ldots j-1}(F(x_{j,t} | x_{1,t} \ldots\ldots, x_{j-1,t})), \theta) \tag{9}$$

Where j represents the number of trees, i represents the number of nodes of each tree, θ represents the parameters set of multivariate copula density function.

For Canonical vine structure, the procedures of the parameters estimation are designed as follows:

Step 1: Considering the characteristic of the credit risk data, calculate the initial values of multivariate Copula density function: $c_{12}((F(x_2),\ F(x_1)),c_{13}((F(x_3),\ F(x_1)),c_{14}((F(x_4),\ F(x_1))$;

Step 2: Using the function $f(x_1, x_2)=c_{12}(F(x_2), F(x_1))\ f(x_1)\ f(x_2)$ and the initial value to obtain $F(x_i|x_1)$:

$$F(x_i \mid x_1) = \frac{\partial C_{i1}(F(x_i), F(x_1))}{\partial F(x_1)},\ i=2, 3, 4 \tag{10}$$

Step 3: Estimating the parameters of $c_{23|1}(F(x_2 \mid x_1), F(x_3 \mid x_1)), c_{24|1}(F(x_3 \mid x_1), F(x_4 \mid x_1))$ by using $F(x_2|x_1)$, $F(x_3|x_1)$, $F(x_4|x_1)$ which obtained in Step 2;

Step 4: Repeating the steps above until all parameters Canonical vine structure pair copula are estimated. The final estimated results are presented in **Table 2**.

Table 2. The parameters of the pair copula with the Canonical vine structure

	Crisis period			Calm period		
	Parameters	S.D	T	Parameters	S.D	T
a_{12}	0.6680	0.1200	5.5670	0.5509	0.0854	6.4511
a_{13}	0.6679	0.1300	5.1377	0.6463	0.0936	6.9053
a_{14}	0.5468	0.1000	5.4680	0.4292	0.0869	4.9385
$a_{23\|1}$	0.3147	0.1678	1.8751	0.3472	0.2400	1.4467
$a_{24\|1}$	0.3551	0.1622	2.1888	0.3100	0.0949	3.2666
$a_{34\|12}$	0.2937	0.1700	1.7275	0.1625	0.0474	3.4288

Note: From July 2006 to August 2008 is the calm period; from September 2008 to December 2010 is the crisis period.

From the significance of parameters, we can conclude that the pair Clayton Copula function with Canonical vine structure passes the *t* test both in crisis period and calm period, therefore, the copula functions with Canonical vine structure can describe the low tail dependency of the credit risk.

4.3. Parameters Estimation of Pair Copula with D Vine Structure

Under the D vine structure, the joint density function of Copula function $f(x_1, x_2, x_3, x_4)$ is as follow:

$$\begin{aligned} f(x_1, x_2, x_3, x_4) = {} & f(x_1) \cdot f(x_2) \cdot f(x_3) \cdot f(x_4) \\ & \cdot c_{12}(F(x_1), F(x_2)) \cdot c_{23}(F(x_2), F(x_3)) \cdot c_{34}(F(x_3), F(x_4)) \\ & \cdot c_{13|2}(F(x_1 \mid x_2), F(x_3 \mid x_2)) \cdot c_{24|3}(F(x_2 \mid x_3), F(x_4 \mid x_3)) \\ & \cdot c_{14|23}(F(x_1 \mid x_2, x_3), F(x_4 \mid x_2, x_3)) \end{aligned} \tag{11}$$

The log-likelihood function of D vine is:

$$\sum_{j=1}^{n-1}\sum_{i=1}^{n-j}\sum_{t=1}^{T}\log(c_{i,j+i|i+1,\dots,i+j-1}(F(x_{i,t} \mid x_{i+1,t},\dots,x_{i+j-1,t}), F(x_{i+j,t} \mid x_{i+1,t},\dots,x_{i+j-1,t})),\theta) \tag{12}$$

Where *j* represents the number of trees, *i* represents the number of nodes of each tree, θ represents the parameter set of pair copula density function.

For D vine structure, the procedures of the parameters estimation are designed as follows:

Step 1: Considering the characteristic of the credit risk data, calculate the initial values of multivariate Copula density function:$c_{12}((F(x_2), F(x_1))$, $c_{23}((F(x_3), F(x_2))$, $c_{34}((F(x_4), F(x_{34}))$.

Step 2: Using the function $f(x_1, x_2)=c_{12}(F(x_2), F(x_1))\ f(x_1)f(x_2)$ and the initial value to obtain $F(x_i|x_{i-1})$:

$$F(x_i \mid x_{i-1}) = \frac{\partial C_{i,i-1}(F(x_i), F(x_{i-1}))}{\partial F(x_{i-1})},\ i=2, 3, 4 \tag{13}$$

Step 3: Estimating the parameters of the second-order pair Copula function $c_{13|2}(F(x_3|x_1), F(x_1|x_2))$, $c_{24|3}(F(x_2|x_3), F(x_4|x_3))$ by using $F(x_2|x_1)$, $F(x_3|x_2)$, $F(x_4|x_3)$ which obtained in Step 2.

Step 4: Repeating the steps above until all the parameters of D vine structure pair copula are estimated. The final estimated results are presented in **Table 3**.

Table 3 The parameters of the pair copula with D vine structure

	Crisis period			Calm period		
	Parameters	S.D	T	Parameters	S.D	T
a_{12}	0.6834	0.2112	3.2355	0.5800	0.1496	3.8775
a_{23}	0.6154	0.1951	3.1538	0.5904	0.1595	3.7013
a_{34}	0.4210	0.1384	3.0419	0.3328	0.0987	3.3707
$a_{13|2}$	0.3266	0.1845	1.7703	0.3608	0.2249	1.6045
$a_{24|3}$	0.3999	0.2333	1.7139	0.3231	0.1831	1.7641
$a_{14|23}$	0.2921	0.1868	1.5635	0.1628	0.1136	1.4334

Note: From July 2006 to August 2008 is the safety period; from September 2008 to December 2010 is the crisis period

Similar to the results of the pair copula with Canonical vine structure, D vine structure copula models also pass the t test both in crisis period and calm period, and we can conclude that the copula functions with D vine structure can describe the low tail dependency of the credit risk.

4.4. Comparison and Analysis of Models

From the above results, we can find that the pair Clayton Copula function have highest goodness of fit. For purpose of getting the optimal decomposition structure, In line with Fermanian (2005), Chen and Fan (2005), Genest, Quessy, and Rémillard (2006), Manner (2007), Nguyen, Bhatti, & Hayat (2014), Chen, Wei, Lang, Lin, & Liu, (2014), we apply AIC and BIC criteria to compare the Canonical vine structure and D vine structure. The statistics of AIC and BIC are shown in Equation (14) and (15):

$$AIC = -2LR + 2K \tag{14}$$

$$BIC = -2LR + K \log T \tag{15}$$

Here LR represents the value of log-likelihood function, K represents the number parameters need to be estimated, T represents sample lengths. According to AIC and BIC criteria, if the smaller the value of AIC and BIC is, the pair copula models are better fitted. The results of comparison are reported in **Table 4**.

Table 4. Comparison of the Canonical vine and D vine structure

		Canonical vine structure Copula	D vine structure copula
Calm period	AIC	−356.5315	−349.5931
	BIC	−344.7098	−337.7714
Crisis period	AIC	−424.0730	−379.1256
	BIC	−412.1391	−367.1917

Judging by the **Table 4**, Canonical vine and D vine multivariate models can describe the characteristic of multivariate credit risk correlation. However, we can find the AIC and BIC of Canonical vine structure model are smaller than those of D vine structure by comparing these models, and both in calm period and the crisis period, the Canonical vine structure copula models are superior to D vine structure models. The possible explanation is that the Canonical vine structure is more suitable for decomposing the data when there is a center node in time series data. In our industry credit risk data, there is a strong cyclical industry, and this industry has a significant effect on other industries. Therefore, in our paper, the Canonical vine structure is more suitable for decomposition of pair copula, and the simulation and calculation of the credit risk VaR will be based on the Canonical vine structure copula in Section 5.

5. The Risk Measurement of Credit Portfolio Based on Pair Copula-VaR

5.1. The Description of Credit Portfolio

This paper chooses a commercial bank's credit data to empirically study the credit portfolio's risk based on Pair Copula VaR. The bank's loan portfolio data are shown in the **Table 5**. In order to compare the risk of credit portfolio between crisis period and calm period, we design two simulation scenarios. Scenario 1 is August 31, 2008 which is considered as the calm period. Scenario 2 is December 31, 2008 which is considered as the crisis period.

Table 5. Summary of the credit portfolio

Scenario	Item	Industry I	Industry II	Industry III	Industry IV
2008-8-31	Loan(thousand)	612129.8	630020.1	165198.0	292275.3
	Percentage	36.02%	37.07%	9.72%	17.20%
	Credit rating	0.841	0.762	0.733	0.851
2008-12-31	Loan(thousand)	123948.07	140913.91	32073.26	57165.81
	Percentage	35.00%	39.79%	9.06%	16.14%
	Credit rating	0.798	0.735	0.691	0.804

It can be seen from the table that the commercial bank made some portfolio adjustments during the sample periods. The bank increased the loan to the II industry and decreased loan to other industries after 2008-8-31. Is the portfolio adjustment decreasing or increasing the total credit risk of the credit portfolio? Our paper attempts to find the answer by using the pair copula VaR models.

5.2. The Parameter Setting for Pair Copula VaR

According to the risk measurement steps of 2.2, some key parameters including confidence level, risk measure cycle, distribution of credit risk data, credit risk exposure, credit ratings, default probability and LGD (loss given default) are need to be set before the simulation.

(1) Confidence level. The confidence level is determined by the individual commercial banks' risk preference. For example, CITI bank chooses 95.4% as its confidence level and J.P.Morgan chooses 95%. In our paper, we choose the confidence level of 99% in line with the standard of the Basel Committee.

(2) Risk measure cycle. The setting of risk measure cycle reflects the dynamic equilibrium of risk management costs and benefits. If the cycle is too short, the risk management costs will be too high and be adverse to commercial bank's operating efficiency. If the cycle is too long, then commercial bank cannot control risk effectively. Banks set the risk measure cycles according to their service types, assets characters and the speed of positions liquidation. This paper chooses 10 trading days (two weeks) as the risk measure cycle. It can reflect the balance relationship between frequent supervision costs and benefits of identify potential risk early.

(3) Distribution characteristics of credit risk. According to the empirical results of Section 3 and Section 4, we use Clayton Copula function under Canonical vine structure to describe the low tail dependency structure when calculating the pair copula VaR.

(4) Credit risk exposure. To simplify the calculation, the risk exposed loans are limited to be mature in 1 year, thus, the total credit risk exposure are the value of principal and interest of the loans to be mature in 1 year.

(5) Measurement of credit rating. Referring to the technical documents of JP Morgan's Metrics and the credit rating method which suggested by scholars, The credit ratings are divided into AAA, AA, A, BBB, BB, B, C and the default rating D. When the value of credit risk p is larger than 0.5, we classify the corresponding enterprise to invest degree (Degree BBB and above). On the contrary, when the value of p is smaller than 0.5, we classify the corresponding enterprise to speculate degree (under the Degree BBB). Then we can also define the credit risk ranges of all credit levels (As shown in Appendix A).

(6) Default probability. We can calculate the credit rating transition probability matrix (as shown in Appendix B) from the default probability data of each list company. We define the enterprises of rating D as the theoretical default enterprises. On the basis of credit rating transition matrix and the equation (16), we can get the default probability of different credit rating for Chinese listed companies.

$$PD = P_{i\text{-}D} \times 25.73\% \tag{16}$$

Where the $P_{i\text{-}d}$ represents the probability of credit rating i transfer to the theoretical default rating D after one year, the i separately represents the degree of AAA, AA, A, BBB, BB, B and C. The estimated default probability results of each credit rating are shown in the **Table 6**.

Table 6. Default probability of different credit rating

Credit rating	AAA	AA	A	BBB	BB	B	C	D
Default probability	0.15%	0.04%	0.21%	0.25%	0.35%	0.78%	3.32%	6.62%

(7) LGD (loss given default). When the LGD of mortgage loan is relatively small, the recovery rate is higher for the credit loan and the guarantee loan with no collaterals will have full amount of the loss if the debtor is default. Referring to the research of Zhang (2004), we define the LGD of

credit loan and guarantee loan as 100%, the LGD of secured loan to 50%.

5.3. Measurement of Credit Portfolio Risk

In order to compare the influences of credit rating on VaR, Our paper distinguishes the actual credit quality and the high credit quality of the portfolio. The actual credit portfolio contains all loans of the sample portfolio, and the high credit quality portfolio contains the assets with the credit rating equal or higher than degree A. We use the pair copula VaR models designed in Section 3, and simulate the changes of the credit rating 10000 times, thus, the VaR of different scenarios of the sample commercial bank are finally acquired in Table 7 and Fig.2.

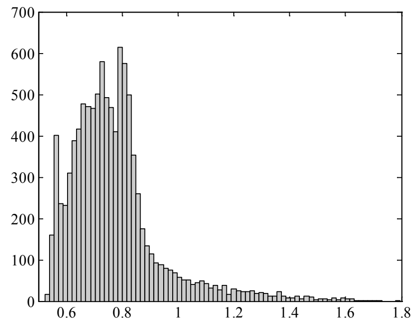
Table 7. Optimal weights of the credit asset and the VaR

Scenario	Credit portfolio	optimal weights				VaR
2008-8-31	High quality portfolio	0.387605	0.155459	0.068151	0.388785	0.001382529
	Actual quality portfolio	0.24291	0.248736	0.313043	0.195311	0.001674174
2008-12-31	High quality portfolio	0.268454	0.250634	0.181746	0.299165	0.001401286
	Actual quality portfolio	0.124148	0.246608	0.165716	0.463527	0.00172049

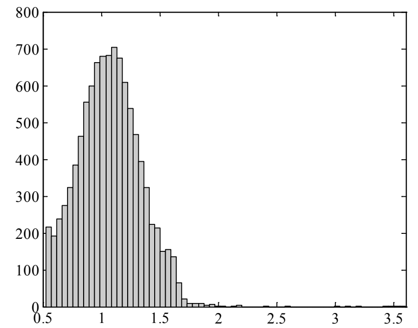
From the Monte Carlo simulation results of credit portfolio VaR, we can see the expected losses of high quality credit portfolio are lower both in crisis period and calm period. Therefore, the first choice for commercial banks to decrease the risk of credit portfolio is to improve the quality of credit assets. But in actual management of the commercial banks, it is very difficult for commercial banks to enhance the credit assets' quality, so the direct way to decrease the credit portfolio risk is to optimize the weights of the credit assets. Comparing to the weights of actual credit portfolio and the optimal weights calculated by the pair copula models, we can provide another option for the sample commercial bank to decrease its credit portfolio risk. Since the optimal weights of the industry III and IV is larger than the actual weights in Table 5, the commercial bank should issue more loans to information technology, bio-medicine and other growth industries. Meanwhile, some weak cyclical industries, such as chemical and plastic industries can also be allocated more loans, especially in crisis period.

6. Conclusion and Implications

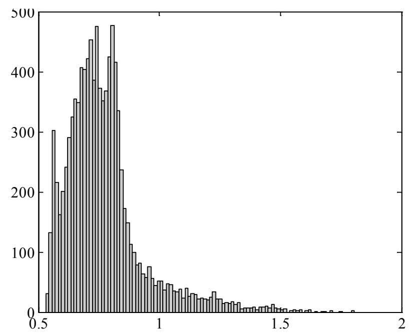
This paper builds a pair copula VaR model to evaluate the credit portfolio risk, the empirical results show that the pair copula VaR model is an effective method since it can capture the high dimensional low tail dependency of the credit assets in portfolio. Based on the specific procedures of estimating the parameters of pair copula and simulating the pair copula VaR provided by the paper, we can find pair Clayton copula is the most appropriate function to describe the high dimensional low tail dependency of the credit assets in portfolio, and the pair copula VaR model can not only evaluate the possible loss of the credit portfolio, but also get the optimal weights of the credit assets in portfolio. Based on the research results, the commercial banks can carry out the following management strategies of credit risk. Firstly, commercial banks should make accurate adjustment of the credit assets allocation based on the optimal weights of the credit assets, thus to decrease their credit portfolio risk. Secondly, commercial banks should improve the credit quality of their assets portfolio. The direct way to decrease the credit portfolio VaR is to enhance the overall quality of the credit assets. Thirdly, on the basis of accumulated data, commercial banks can develop Pair Copula VaR model and other advanced models which are more suitable for the own characteristics of banks, thus to find the best method to predict and control the commercial banks' risk.



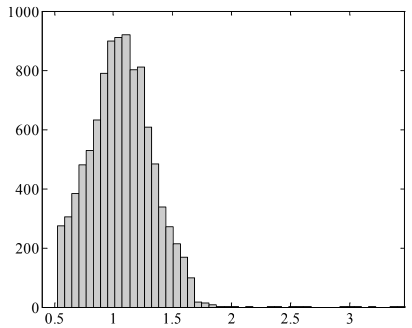
(a) Expected loss of high quality portfolio in calm period (frequency)



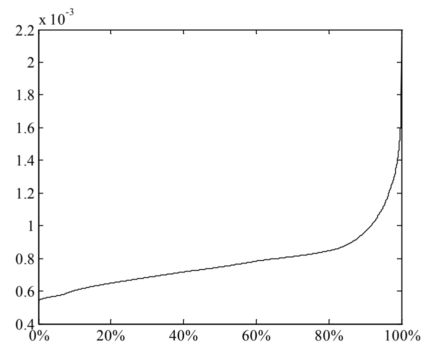
(b) Expected loss of actual quality portfolio in calm period (frequency)



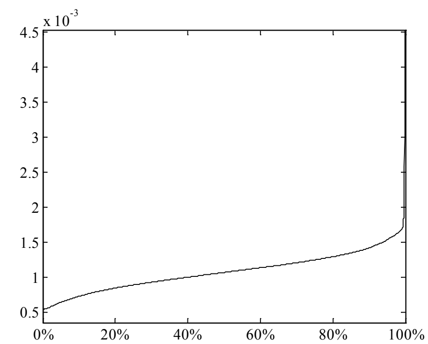
(c) Expected loss of high quality portfolio in crisis period (frequency)



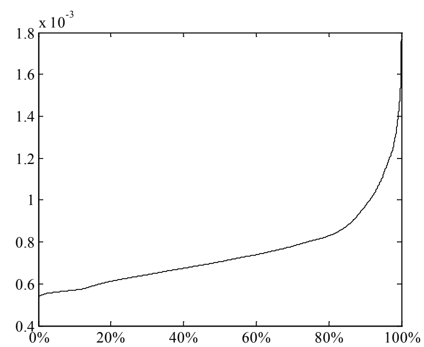
(d) Expected loss of actual quality portfolio in crisis period (frequency)



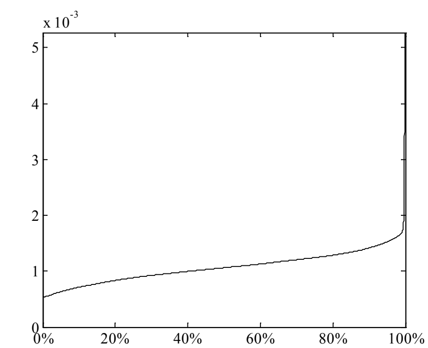
(e) Expected loss of high quality portfolio in calm period (quantile)



(f) Expected loss of high quality portfolio in crisis period (quantile)



(g) Expected loss of high quality portfolio in calm period (quantile)



(h) Expected loss of actual quality portfolio in crisis period (quantile)

Fig.2. Monte Carlo simulation of the credit portfolio

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Appendix

Appendix A. The standard of the credit rank

Credit rank	Range	Percentage	Credit rank	Range	Percentage
AAA	$0.9<=p<=1$	4.30%	BB	$0.4<=p<0.5$	16.70%
AA	$0.8<=p<0.9$	9.50%	B	$0.3<=p<0.4$	8.00%
A	$0.65<=p<0.8$	29.70%	C	$0.2<=Z<0.3$	2.20%
BBB	$0.5<=p<0.65$	28.20%	D	$0.0<=Z<0.2$	1.40%

Appendix B. The credit rank transition probability matrix(1990~2010)

Rank(T)	Rank(T+1)							
	AAA	AA	A	BBB	BB	B	C	D
AAA	38.98%	22.06%	15.12%	8.77%	6.52%	5.87%	2.10%	0.58%
AA	17.37%	28.24%	24.54%	12.08%	10.14%	7.42%	0.06%	0.14%
A	4.99%	12.46%	34.88%	19.89%	17.03%	8.98%	0.95%	0.83%
BBB	2.02%	3.39%	21.99%	32.12%	28.28%	9.66%	1.54%	0.99%
BB	0.71%	2.53%	10.81%	24.13%	40.62%	17.98%	1.87%	1.35%
B	2.93%	2.10%	6.59%	9.99%	25.66%	37.49%	12.19%	3.05%
C	3.71%	1.14%	2.69%	8.61%	18.30%	28.87%	23.79%	12.89%
D	2.18%	1.73%	0.70%	7.84%	12.41%	20.10%	29.31%	25.73%