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Investigating Robust Estimation and Forecasting of Volatilities of Futures with Interquartile Range Models

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Abstract

A robust proxy of volatility, interquartile range is investigated with the estimation and forecasting of volatility of five American futures using GARCH-type models. With utilizing realized volatility as the yardstick of true underlying volatility, the Mincer-Zarnowitz (MZ) regression and four loss functions in Hansen and Lunde (2005) are employed as criterions for assessing the forecasting ability of competitive volatility models, both in sample and out of sample. It is found that, in samples of NY Light Crude (CL) and NY Natural Gas (NG) which are more volatile and have more extreme outliers than the other three, interquartile range models outperforms those standard models. But in samples of Dow Futures (DJ), Nasdaq 100 Futures (ND) and S&P 500 Futures (SP) which are relatively stable, the results are, however, opposite.

JEL Classifications: F3, G1, C5

Keywords: GARCH model, interquartile range, realized volatility, Mincer-Zarnowitz regression, loss function

1. Introduction

1.1. Volatility Models and the Range of Prices

Volatility has been extensively discussed in the literatures of financial economics and econometrics. As a measure of risk, volatility plays a key role in the areas of asset pricing, portfolio allocation and risk management. Hull and White (1987) extend the Black and Sholes (1973) option valuation model by incorporating the time-varying characteristic of volatility. In portfolio allocation such Greeks as delta, gamma, theta, vega and rho are all used for measuring different degrees of risk, and all of them are related to volatility. Volatility models with range also have their roots in financial literature. Chou (2005) proposes the CARR (Conditional Autoregressive Range) model to forecast volatility of assets' prices utilizing ranges as the proxy of volatility. Empirical results reveal that CARR models significantly outperform GARCH models in forecasting. However, it still needs further verification that whether range is a proper proxy of volatility. Beckers (1983) argues that adopting range as the proxy of volatility has some problems. First of all, when trades occur infrequently, the range estimator may have a significant downward bias; secondly, ranges are highly sensitive to outliers and it also results bias; thirdly, the highest and lowest prices used to construct range may reflect behaviors of disadvantaged traders but not general traders, hereby it may be not a reliable indicator of asset's value. Alizadeh, Brandt and Diebold (2002) point out that the highest prices observed everyday are actually half spread higher than the real highest prices, and the lowest prices observed everyday are half spread lower than those lowest ones. Hence the ranges constructed by the observed highest and lowest prices are different from the real ranges, it is appropriate to consider inter-quantile ranges as useful proxies of realy ranges.

1.2. Ranges Models and its Virtues

Literatures of using range as the proxy of volatility are prolific. According to the definition of Chou (2005), range is the distance between the highest and the lowest prices of assets in some fixed sampling period. Mandelbrot (1963) uses range to test the long-run dependence characteristic of asset prices. Parkinson (1980) argues the natural logarithm of stock prices roughly follow Random Walk process with a constant diffusion parameter which equals the variation of returns. He also compared variations of the range estimator with variations of the traditional return estimator and found that the range estimator is five times more efficient than the return estimator. Wiggins (1991) shows that compared with the return estimator, the range estimator have a problem of downward bias. And its efficiency is damaged a lot by the outliers. Once outliers get removed from the sample, the range estimator remains high efficiency. Furthermore, Andersen and Bollerslev (1998) find that range estimator provides a higher R-square value than the traditional return estimator in Mincer-Zarnowitz (MZ) regression when using realized volatility as the proxy of true underlying volatility.As to the dynamic structure of time-varying volatility, Engle (1982) first introduces ARCH model for the volatility clustering. Then Bollerslev (1986) generalizes it as GARCH model. Surveys like Bollerslev, Chou, and Kroner (1992) provide many explanations and applications of the family of GARCH models in economics and finance. As to the dynamic structure of time-varying volatility, Engle (1982) first introduces ARCH model for the volatility clustering. Then Bollerslev (1986) generalizes it as GARCH model. Surveys like Bollerslev et al. (1992) provide many explanations and applications of the family of GARCH models in economics and finance. Ding, Granger and Engle (1993) argues that the absolute return rates $|r_t|$ have significant autocorrelations comparing to the returns, and transformations of the absolute return rates like $|r_t|^d$ also have this feature. As d approaches 1, the autocorrelations become more and more significant，a feature by the name of long memory in financial literatures. They also use Box-Cox transformation mode in volatility equation and constructed APARCH (Asymmetric power ARCH) model. This model not only proposes the concept of volatility asymmetry but also includes seven different types of ARCH models. It is the first step to construct the family of ARCH models. Hentschel (1995) introduces the family of GARCH models and constructs a more general model. Recently literatures have incorporated ranges already. Lin and Rozeff (1994) incorporate ranges into the volatility equation of GARCH model. Alizadeh et al. (2002) demonstrate that the logarithmic range is highly efficient, has an approximately normal distribution, and is not contaminated by the market microstructure. They build a two-factor SV model with the logarithmic range: one is a highly persistent factor and the other is a mean-reverting factor. Brandt and Jones (2006) combine the logarithmic range with EGARCH model in the fractionally integrated pattern, and significantly improved the correctness of out-of-sample forecast with S&P500 index.

1.3. Range Based GARCH Type Models

Chou (2005) proposes a range based volatility model by the name of CARR model (Conditional Autoregressive Range model), using GARCH structure to capture the dynamics of conditional range. CARR model has traits as the following: 1). Quasi-maximum Likelihood Estimation of the model delivers consistent estimation; 2). CARR model is much easier to manage empirical study than many other volatility models because it can be estimated by the same statistical software for

GARCH models. One important extension of CARR model is ACARR model by Chou(2005) which disentangles the asymmetry in financial markets into three types: trending behavior, skewness persistence and the interaction of the two. ACARR model decomposes ranges into upward ranges and downward ranges to better capture the asymmetry. His empirical study with S&P500 index shows that ACARR model performs better than traditional CARR models. Another important extension of CARR model is TCARR model (Threshold CARR) of Lin (2005) who adopts past ranges or other exogenous variables as threshold variables to depict the asymmetry in financial markets by the method of regime switching. Via a Bayesian approach, Lin (2005) employs MCMC (Markov Chain Monte Carlo) method to estimate the parameters iteratively. Chou, Wu, and Liu (2004) combine CARR model with DCC (Dynamic Conditional Correlation) model by Engle (2002) and develop DCC-CARR model. These models all use ranges as the proxy of volatility and gain important improvement empirically.

2. Method

2.1. Econometric Methodology

According to the definition in Chou (2005), P_t represents the logarithm of asset price at time t. The range is defined as $R_t = Max\{P_\tau\} - Min\{P_\tau\}$, and $\tau = t-1, t-1+1/n, t-1+2/n, ..., t$ from the sampling period $t-1$ to t. As n grows, the time interval becomes shorter, and the measurement of ranges is more accuracy theoretically. $Max\{P_\tau\}$ represents the highest price of all observable prices from the sampling period $t-1$ to t, which is form yesterday to today in our paper. In the similar way, $Min\{P_\tau\}$ represents the lowest of all observable prices in a day. The linear structure of CARR (p, q) can then be represented as follows:

$$R_t = \lambda_t \varepsilon_t, \lambda_t = \omega + \sum_{i=1}^{p} \alpha_i R_{t-i} + \sum_{j=1}^{q} \beta_i \lambda_{t-i}, \varepsilon_t \mid I_{t-1} \sim f(1, \xi_t) \tag{1}$$

Where $\lambda_t \equiv E[R_t \mid I_{t-1}]$ represents the conditional range under information set until time t-1. ε_t is the disturbance with identical distributions and it can also be regarded as the standardized range, which is $\varepsilon_t = R_t / \lambda_t$. Generally ε_t follows a non-negative distribution with unit mean. $\omega > 0$ is the inherent uncertainty of the dynamic process and it can also be interpreted as the initial level of the range. Those $\alpha_i > 0, i = 1, ..., p$ are coefficients of the lag ranges. They measure the short-run effects. Similarly, those $\beta_i \geq 0, j = 1, ..., q$ measure the long term effects. Plus, in order to satisfy the stationary condition, it requires that $\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_i < 1$.The unconditional expectation is then $\overline{\omega} = \omega / (1 - (\sum_{i=1}^{p} \alpha_i + \sum_{j=1}^{q} \beta_i))$. More about the CARR model see Chou (2005).

Because range is defined as the distance of highest and lowest prices every day, it is hypersensitive to the outliers. Therefore CARR model actually suffers from the critique of exaggerating volatility. In response, we adopt interquartile ranges as the proxy of volatility to construct a more robust form of CARR model. The definition of the interquartile range is:

$$R_t(H_i, L_j) = Quantile(100-i)\{P_\tau\} - Quantile(j)\{P_\tau\} \tag{2}$$

Where $\tau = t-1, t-1+1/n, t-1+2/n, ..., t$, and $i = 0\% \sim 50\%, j = 0\% \sim 50\%$. In the equation above, $Quantile(j)\{P_\tau\}$ represents the jth percentile of prices in a unit time interval (such as a day or a week). For instance $Quantile(0\%)\{P_\tau\}$ is the lowest price of all observable prices from yesterday's close price to today's close price, and $Quantile(100\%)\{P_\tau\}$ is the highest one. With the notation above, we renew the definition of standard range as:

$$R_t = Quantile(100\%)\{P_\tau\} - Quantile(0\%)\{P_\tau\} = R_t(H_{0\%}, L_{0\%}) \tag{3}$$

In this case range is a special case of interquartile range. Besides, if $i = j$, we call such an interquartile range as symmetric. And if $i \neq j$, we call it asymmetric interquartile range. We select 14 percentiles in the following study. They form 49 groups of combination, with 7 groups of symmetric interquartile ranges and 43 groups of asymmetric ones.

2.2. Data and Relevant Measures

The data is consisted of five different sets: NY Light Crude (CL), Dow Futures (DJ), Nasdaq 100 Futures (ND), NY Natural Gas (NG) and S&P 500 Futures (SP). The data contain observations from January 1, 1998 to March 31, 2004. After deleting the days of insufficient information, the number of daily observations are 1545 (CL), 1570 (DJ), 1570 (ND), 1555 (NG) and 1570 (SP). According to Alizadeh et al. (2002), future prices are generated by open outcry and futures markets usually settle all the unbalanced accounts in settlement days. Compare to spot prices, future prices is a better proxy of true market prices. The data resource is coming from TickWrite database.

In the calculation of realized volatility, Hol and Koopman (2002) propose two methods. First of all, the intraday returns and overnight returns are defined as follows:

$$r_{t,d} = 100(P_{t,d} - P_{t,d-1}), \quad r_{t,N} = 100(P_{t,o} - P_{t,D}) \tag{4}$$

$r_{t,d}$ represents the dth intraday return in the five minutes frequency at day t, and $r_{t,N}$ represents the overnight return at day t. $P_{t,o}$ represents the log open price at day t. $P_{t,D}$ represents the log close price at day t-1, or the Dth price in the five minutes frequency at day t-1. For instance, in Nasdaq 100 Futures there are 78 intraday returns in one day, so D=78. Then we define the realized volatility:

$$RV_{t,1} = r_{t,N}^2 + \sum\nolimits_{d=1}^{D} r_{t,d}^2, \quad RV_{t,2} = (1+c)\sum\nolimits_{d=1}^{D} r_{t,d}^2 = (\sigma_{OC}^2 + \sigma_{CO}^2)\sum\nolimits_{d=1}^{D} r_{t,d}^2 / \sigma_{OC}^2 \tag{5}$$

The realized volatility of one day in the above equation is directly calculated by adding the sum of squared intraday returns with the squared overnight returns. The stock and commodity markets trade in a different way from the exchange markets, and the overnight returns of those markets could be more volatile than the intraday returns. Therefore it is probably not good to add them together directly. We use another method to calculate the synthesized realized volatility consisting of observable component in the daytime and the unobservable component during the night with adjusted scales: $\sigma_{OC}^2 = \mathrm{var}(\sum\nolimits_{d=1}^{D} r_{t,d})$ represents the variation of all the intraday returns sums, and $\sigma_{CO}^2 = \mathrm{var}(r_{t,N})$ represents the variation of all the overnight returns. We use equation (1.5) to calculate realized volatility in our paper. In our empirical study the number of trades $D = 78$ and the total in a day is then $M = 228$. If $1 + c \neq M / D = 3.69$, explanation by Hansen and Lunde (2005) states: besides the sampling error, intraday returns have strong autocorrelations, resulted mainly by bid-ask bounces. Also volatility appears more intense in nighttime than that in daytime.

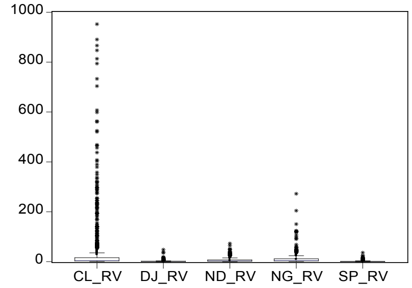

Figure 1. The box-plot of Realized Volatility in CL, DJ, ND, NG and SP

Now we use realized volatility calculated according to equation (1.5) to see some basic characters about the five futures' volatility. Through the box-plot in figure 1, it is easy to see that the realized volatilities of the samples NY Light Crude (CL) and NY Natural Gas (NG) include many outliers with high values. The difference of realized volatilities illustrates that the commodity futures (CL, NG) and finance futures (DJ, ND, SP) are different in nature of their volatility. In order to know whether CARR model with interquatile ranges can perform consistently with data sets of totally different volatilities, we divide the data sets into two parts: (1) NY Light Crude (CL), NY Natural Gas (NG) and (2) Dow Futures (DJ), Nasdaq 100 Futures (ND), and S&P 500 Futures (SP).

3. Empirical Study and Results

We use 49 interquartile ranges CARR models in the estimation. In the selection of orders, Chou (2005) uses LR (likelihood ratio) test to decide the orders of specification, and concludes that the CARR (1, 1) model is adequate for most cases. Hansen and Lunde (2005) also points out that models with less lags usually perform better, even if the parameters estimated of the latter's lags is significant. Hence we follow Chou (2005) and Hansen and Lunde (2005) and employ CARR (1, 1) model for all the interquartile ranges. For there is actually no significant difference between symmetric and asymmetric interquartile ranges in the estimations, we select 7 symmetric ones to present their estimation results. This shows CARR model can capture most autocorrelations of primitive data. Further we use both in-sample and out-of-sample forecast tests to see whether our models are really robust to outliers.

3.1. In-Sample Forecasts

Provide dates defining the periods of recruitment and follow-up and the pnmary sources of the potential subjects, where appropriate. If these dates differ by group, provide the values for each group. Chou (2005) and Brandt and Jones (2006) both use the Mincer-Zarnowitz (MZ) regression to compare in-sample forecast:

$$MV_t = a + b * FV_t \,(\text{CARR with interquartile range}) + u_t \tag{6}$$

We employ realized volatilities of five minutes as the MV_t (measured volatility), and fitted values of different CARR models with interquartile ranges as FV_t (forecasted volatility). We made 1400 times of in sample forecast. After adjusting MV_t and FV_t in scales, if the coefficient a is not significantly different from zero and the coefficient b is not significantly different from one, it means the estimation via CARR models with interquartile range is unbiased. Adjusted R-squared value is also used as the criterion of assessing forecast power of different models. Newey-West method[1] is used in MZ equation to be against the heteroskedasticity and autocorrelation. We can see from table 5 and 6 that (1): In the samples with high volatility like CL and NG, $H_{5\%}-L_{30\%}$ is always the best interquartile range for CARR model. Further a careful scrutiny reveals that the seven best groups of interquartiles in CL and NG do share a same feature that they exclude more low prices than high prices. It perhaps means that when volatility is high, investors in the markets concern more about how high the prices can attain other than those low prices. (2) In the samples with low volatilities like ND, DJ and SP, the results seem to be more ambiguous. First of all, the best seven groups of interquartile ranges in DJ and SP have significant different features from those in CL and NG. They are all symmetric but not like those in CL and NG, which are most asymmetric ones. Additionally the standard ranges CARR models in DJ and SP are respectively the third-best and the first-best. Others in the best seven groups are also similar to the standard CARR model. It looks like that the standard CARR model seems to be already adequate for the samples with low volatility, but for samples with high volatility and many outliers, an asymmetric interquartile CARR model performs better.

3.2. Out-of-Sample Forecasts

Analysis of data and the reporting of the results of those analyses are fundamental aspects of the conduct of research. Accurate, unbiased, complete, and insightful reporting of the analytic treatment of data (be it quantitative or qualitative) must be a component of all research reports. Researchers in the field of psychology use numerous approaches to the analysis of data, and no one approach is uniformly preferred as long as the method is appropriate to the research questions being asked and the nature of the data collected. The methods used must support their analytic burdens, including robustness to violations of the assumptions that underlie them, and they must provide clear, unequivocal insights into the data. In this section, we use out of sample forecasts to compare different models. Multi-periods out-of-sample forecasts are made iteratively as follows:

$$\lambda^{f}_{T,T+k} = \hat{\omega} + (\hat{\alpha}_1 + \hat{\beta}_1)\lambda^{f}_{T,T+k-1} \tag{7}$$

For the model assessment, we continue to use the loss functions in Hansen and Lunde (2005), the first two loss functions are regular MSE and MAE. The third QLIKE function is proposed by Bollerslev (1996). It is called Gaussian quasi-maximum likelihood function, stemmed from the likelihood function of GARCH model. R2LOG is proposed by Pagan and Schwert (1990). The forecasted values through CARR models usually have a scale difference from realized volatilities. Brandt and Jones (2006) argue that Mincer-Zarnowitz regression and the loss functions are not capable of telling us whether the difference of forecast power is significant in statistics. They employed a T-test for this purpose; see the appendix for more details. We set standard range CARR model as model p, and interquartile range CARR model as model q. Then we are able to test whether the employment of interquartile improves our estimations. Before calculating the loss function, we have to adjust them according to their different scales:

$$MV_{1+r,T+r} = \phi_r * FV_{1+r,T+r} + u_{1+r,T+r}, AFV_{T+k+r} = \widehat{\phi}_r * FV_{T+k+r}, r = 0,1,2,...,n-1 \tag{8}$$

In table 1, we listed all the results of comparison of interquartile range CARR models with standard range CARR models of both in sample and out of sample forecasts. The results are very consistent. And out of sample periods forecasts do not show significant difference from each other.

[1] Newey and West (1987) have proposed a more general covariance estimator that is consistent in the presence of both heteroskedasticity and autocorrelation of unknown form.

Table 1. Winning rates of both in-sample and out-of-sample forecasts

In-sample	Out-of-sample					
Adj. R-squared	periods	MSE	MAE	QLIKE	R2LOG	T-test
CL-58%	1	53%	54%	54%	54%	46%
	2	52%	56%	54%	54%	48%
	3	54%	58%	54%	52%	48%
	4	56%	58%	56%	54%	50%
CL-69%	1	65%	65%	67%	65%	33%
	2	79%	79%	81%	79%	50%
	3	77%	78%	81%	75%	46%
	4	79%	75%	79%	77%	44%
DJ-4%	1	0%	0%	0%	0%	0%
	2	0%	0%	0%	0%	0%
	3	0%	0%	0%	0%	0%
	4	0%	0%	0%	0%	0%
ND-85%	1	4%	4%	10%	8%	4%
	2	6%	6%	8%	8%	4%
	3	4%	4%	8%	4%	4%
	4	4%	4%	4%	4%	0%
SP-0%	1	0%	0%	0%	0%	0%
	2	0%	0%	0%	0%	0%
	3	0%	0%	0%	0%	0%
	4	0%	0%	0%	0%	0%

Discription: Winning rates in this table show how many interquartile range CARR models win the standard CARR model among all 48 groups of interquartile range CARR models.

4. Discussion and Concluding Remards

Range as a proxy of volatility is highly efficient, but it also suffers from critiques of Hypersensitivity to outliers as pointed by Beckers (1983), Chou (2005a), and Alizadeh et al. (2002). With the idea of employing a more robust measurement of range, we combine CARR model in Chou (2005) with interquartile range to moderate the damage of outliers in the estimation of volatility.

The empirical study based five types of U.S. commodity and financial futures discerned two situations. One type is commodity futures who have high volatility and many outliers like NY Light Crude (CL) and NY Natural Gas (NG). Ranges of these assets exaggerate their volatilities and CARR models with interquartile range performs better than the standard range CARR models when realized volatility is employed as the measure of underlying volatility. Especially CARR models with interquartile ranges are identified as the best performer among 49 groups of both symmetric and asymmetric interquartiles for the commodity futures, via both in sample and out of sample forecasts.

The other type is financial futures who have mild volatility and few outliers like like Dow Futures (DJ), Nasdaq 100 Futures (ND) and S&P 500 Futures (SP). CARR model with standard range is already adequate enough. No other interquartile range CARR models can outperform it significantly according to the test proposed by Brandt and Jones (2005).Theories like EVT emphasize modeling outliers in trading in order to better capture the reality. Here we use an opposite approach by deleting outliers from samples to moderate their damage. Via empirical study we find it works. The reason still remains unknown and worth further studying. Distinguishing outliers with information from those with no information may help in the future studies.

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Appendix

The four loss functions and T-test errors employed by out-of-sample forecast are calculated by:

$$MSE = n^{-1}\sum_{r=1}^{n}(MV_{T+1+r} - AFV_{T+1+r})^2$$

$$MAE = n^{-1}\sum_{r=1}^{n}| MV_{T+1+r} - AFV_{T+1+r} |$$

$$QLIKE = n^{-1}\sum_{r=1}^{n}((\ln(AFV_{T+1+r}) + MV_{T+1+r} / AFV_{T+1+r})$$

$$R^2 LOG = n^{-1}\sum_{r=1}^{n}(\ln(MV_{T+1+r} / AFV_{T+1+r}))^2$$

$$\varepsilon_{p,T+1+t} = MV_{T+1+t} - AFV_{p,T+1+t}$$

$$\varepsilon^2_{q,T+1+t} - \varepsilon^2_{p,T+1+t} = u_{p,q} + \eta_{T+1+t}$$

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